

On the Economics of Information

with Incomplete Markets

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1. Introduction

The literature on equilibrium under incomplete information when direct information gathering possibilities exist has, to date, disclosed two phenomena through which this activity gives rise to externalities and potential inefficiencies. Each of these can be either beneficial or harmful, the net effect of improved information becoming an empirical matter. The first (see Hirshleifer (1971)), is that better information will make the equilibrium prices more variable over time, and therefore, unless the information is of direct value in production or intertemporal consumption decisions, it will reduce average utility whenever risk aversion is present. The second effect (see Green (1973)) is a result of the fact that the price system itself can be a useful proxy for information that is not observed directly. When an individual decides to become informed directly, at some private cost, he fails to take into account the influence of his decision on the way that equilibrium prices depend upon the available information and the resulting improvement in the quality of the information received by those who remain in the uninformed category. As in the case of the Hirshleifer effect, the resulting change in social welfare is indeterminate. The improved information can be of value only if it is useful in resource allocation decisions; otherwise, the demands and supplies of the uninformed group will become still more sensitive to prices, resulting in an even more variable equilibrium price system and a reinforcement of the Hirshleifer effect.

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From these results one might draw the conclusion that, unless information has some real productive value, its optimal level is zero. If no one were informed, futures prices would be the same in every iteration of the market, and then, in the presence of complete contingent claims markets, an optimal pattern of risk-bearing would be achieved. In this paper I introduce a third consideration. Information may be of positive social value even when it is useless for improving private resource allocation decisions. The phenomenon is only operative when there are incomplete markets for contingent claims. It will be shown via an example that the positive value of the information attained through this mode may overcome the negative (direct) Hirshleifer effect and its further reinforcement due to the dissemination of the superior information through the price proxy.

Simply stated, the idea is this: With incomplete futures markets individuals will be trading contracts to hedge against the variability of their incomes. If the payoff to holding futures contracts is not linearly related to the traders' exogenous random incomes, such hedging can not eliminate all of the fluctuation in net returns. If, however, futures prices were to become more variable because of more precise information becoming available to the informed group, the resulting correlation between the value of futures contracts and exogenous income could become closer to unity. Hedging opportunities would improve and the resulting allocation would more closely approximate that of a complete markets equilibrium. This kind of second-best improvement in the risk-spreading potential of futures trading may outweigh the social costs of the additional futures price fluctuations that would result.

In section 2 an example is presented demonstrating the effect under consideration. Section 3 presents a general statement of the problem and

relates this effect to the recent result of Radner and Stiglitz (1975). Unfortunately a complete categorization of valuable information structures has not been attained. We discuss the difficulties involved in this section. A brief summary follows in section 4 .

2. Example

In this section we present a simple partial equilibrium example in which information has a positive social value even though it is not useful for private resource allocation decisions. We then classify, within this context, a class of cases in which information will be unambiguously harmful.

We consider an agricultural commodity which is produced under perfectly competitive conditions by risk averse producers. The number of producers is determined by long-run forces and will be held fixed, temporarily, in the present analysis. The sequence of economic actions is as follows: The crop is planted; the available information is revealed to those agents (producers or others) who have invested in acquiring it; a futures market meets in which contracts for delivery after the harvest are traded; the size of the harvest becomes known to each producer; and, finally, a spot market meets in which the commodity is allocated to the consumers (the demand curve is assumed to be fixed) and the commitments made regarding future delivery are honored. (We ignore bankruptcies and resulting unfulfilled contracts). For simplicity we will assume in this section that everyone receives the information. We ask whether this type of public information is socially valuable. This assumption dissociates the problem from the phenomenon of information transmission through prices mentioned above.

There are three possible levels for the output of each producer and the output of all the producers are perfectly correlated. The possible levels of aggregate output are denoted y_1 , y_2 and y_3 respectively. We will assume that they are equally likely and that the variable costs of production are zero.

If the demand curve is given by

$$(2.1) \quad p = D(y)$$

the three possible prices on the spot market are p_1 , p_2 and p_3 , corresponding to the three levels of output respectively. Let

$$(2.2) \quad v_i = p_i y_i = D(y_i) y_i \quad i = 1, 2, 3$$

be the level of total revenue.

When the futures market meets, a price p_0 will be determined. The problem of the producer will be to choose his purchases of futures contracts, z , to maximize

$$(2.3) \quad E u(v+z(p-p_0)) = \pi_1 u(v_1+z(p_1-p_0)) + \pi_2 u(v_2+z(p_2-p_0)) + \pi_3 u(v_3+z(p_3-p_0))$$

where π_1 , π_2 and π_3 are his beliefs about the probabilities of the three events given the information. In equilibrium the price of the futures contract p_0 will reflect the information available. The other participants in the futures market are assumed to be risk-neutral speculators. They may alternatively be supposed to exist in sufficiently large numbers so that p_0 is kept identically equal to the expected value of the spot price given the existing information. One implication of this is that since the speculators will be indifferent to their level of trading in futures contracts, any desired volume of trading by producers will be compatible with the equilibrium prices determined in this way.

We consider two alternative information structures. The first is "no information", so that $p_0 = E p = E D(y)$. In the second, a signal, β , is received by all individuals. There are two possible values for β , β_1 or β_3 . The informational content of β can be inferred from the rules through which it is generated. If $y = y_1$ then $\beta = \beta_3$ and if $y = y_3$ then $\beta = \beta_1$, with probability one. If $y = y_2$, however, then $\beta = \beta_1$ or β_3 with probability $\frac{1}{2}$ each. That is, if $\beta = \beta_1(\beta_3)$ the informed group knows that $y_1(y_3)$ and hence $p_1(p_3)$ will not come to pass. But they cannot rule out y_2 and p_2 . Formally, the information structure is defined by the likelihoods

$$\begin{aligned}
 (2.4) \quad & h(\beta = \beta_3 \mid y_1) = h(\beta = \beta_1 \mid y_3) = 1 \\
 & h(\beta = \beta_3 \mid y_3) = h(\beta = \beta_1 \mid y_1) = 0 \\
 & h(\beta = \beta_1 \mid y_2) = h(\beta = \beta_3 \mid y_2) = \frac{1}{2}
 \end{aligned}$$

Using Bayes' theorem, the posterior probabilities for prices given β are given by

$$\begin{aligned}
 (2.5) \quad & g(p = p_1 \mid \beta_1) = g(p = p_3 \mid \beta_3) = 0 \\
 & g(p = p_2 \mid \beta_1) = g(p = p_2 \mid \beta_3) = 1/3 \\
 & g(p = p_3 \mid \beta_1) = g(p = p_1 \mid \beta_3) = 2/3
 \end{aligned}$$

Thus, assuming further that

$$(2.6) \quad \frac{p_1 + p_3}{2} = p_2$$

we have that, given this information structure, equilibrium futures prices must be

$$\begin{aligned}
 (2.7) \quad & p_0(\beta_1) = \frac{2p_3 + p_2}{3} \\
 & p_0(\beta_3) = \frac{2p_1 + p_2}{3}
 \end{aligned}$$

Without information, producers choose z to maximize

$$(2.8) \quad 1/3 \left[u(v_1+z(p_1-p_0)) + u(v_2+z(p_2-p_0)) + u(v_3+z(p_3-p_0)) \right]$$

because only one equilibrium p_0 will arise and $\pi_1(p_0) = 1/3$ in the absence of information. This becomes

$$(2.9) \quad 1/3 \left[u(v_1+z(p_1-p_2)) + u(v_2) + u(v_3-z(p_1-p_2)) \right]$$

since $p_0 = E p = p_2$.

With information the producers may choose different values of z when the signal changes. It should be noted that this is not due to a shift in the relative profitability of speculative activity in the two cases. Futures trades will always have zero expected value; they alter their level of purchases because the new information changes the direction in which it is necessary to buy futures in order to hedge risks effectively. When $p_0 = p_0(\beta_1)$ the individual knows that (p_1, v_1) cannot happen and must choose $z(p_0(\beta_1))$ to solve

$$(2.10) \quad \max. \quad 1/3 u(v_2+z(p_2 - \frac{2p_3+p_2}{3})) + 2/3 u(v_3+z(p_3 - \frac{2p_3+p_2}{3})) .$$

Since the futures contract offers an actuarially fair bet, this is maximized for any concave u , where the two realizations are equalized, that is, where

$$(2.11) \quad z(p_0(\beta_1)) = \frac{v_3 - v_2}{p_2 - p_3} .$$

Similarly, if $p_0 = p_0(\beta_3)$, the solution is

$$(2.12) \quad z(p_0(\beta_3)) = \frac{v_1 - v_2}{p_2 - p_1} .$$

This policy results in an overall expected utility of

$$(2.13) \quad 1/2 \left[u\left(\frac{2v_1 + v_2}{3}\right) + u\left(\frac{2v_3 + v_2}{3}\right) \right].$$

We must show only that the value of (2.9) for the optimal choice of z is less than the value of (2.13) for some suitably chosen numbers v_1 , v_2 and v_3 . In this way the beneficial effect of information on producers' hedging possibilities will have been shown to outweigh the Hirshleifer effect of a lower attainable average utility due to the fact that p_0 is not a constant. The simplest way of insuring this is to take

$$(2.14) \quad v_1 = v_3 \neq v_2$$

in which case the result will hold for any concave u as follows:

Since $v_1 = v_3$, the maximum in (2.9) is obtained at $z = 0$ and its value is

$$(2.15) \quad 2/3 u(v_1) + 1/3 u(v_2).$$

Setting $v_1 = v_3$ in expression (2.13) we obtain

$$(2.16) \quad u\left(\frac{2v_1 + v_2}{3}\right).$$

Since $v_1 \neq v_2$ (2.16) exceeds (2.15) for any concave u .

We have seen above that $v_1 = v_3 \neq v_2$ is a sufficient condition for the superior hedging effect to dominate the Hirshleifer effect for all concave utility functions. It is easy to show within the context of this example, that if this

condition is violated, then there exists some concave u for which the improvement in information is harmful.

We turn therefore to the converse question: What is the class of triples, (v_1, v_2, v_3) , such that the Hirshleifer effect predominates for all concave u . With information, as described above, an expected utility of

$$(2.17) \quad \frac{1}{2} u\left(\frac{2v_3 + v_2}{3}\right) + \frac{1}{2} u\left(\frac{2v_1 + v_2}{3}\right)$$

can be attained.

Without information, since p_2 is the expected price, one can achieve the expected utility

$$(2.18) \quad \frac{1}{3} u(v_2) + \frac{2}{3} u\left(\frac{v_1 + v_3}{2}\right)$$

To show that the information is harmful for all risk averters, it is necessary and sufficient to show that there is a mean-preserving spread of the random variable giving rise to the utility (2.18) which dominates that giving rise to (2.17). (See Rothschild and Stiglitz (1971)). Since the mean payoff is the same in these two cases, this is therefore equivalent to finding one that converts one payoff distribution into the other exactly.

When the outcome is v_2 , we perturb this with weights β and $1-\beta$ so that it takes the values $\frac{2v_1+v_2}{3}$ and $\frac{2v_3+v_2}{3}$, and that the mean is unchanged; similarly, let α and $1-\alpha$ be the weights in the mean-preserving spread of $\frac{v_1+v_3}{2}$ into these two points.

The values of α and β are given by

$$(2.19) \quad \alpha \left(\frac{2v_1 + v_2}{3}\right) + (1 - \alpha) \left(\frac{2v_3 + v_2}{3}\right) = \frac{v_1 + v_3}{2}$$

$$(2.20) \quad \beta \left(\frac{2v_1 + v_2}{3}\right) + (1 - \beta) \left(\frac{2v_3 + v_2}{3}\right) = v_2$$

In order for α and β to define a mean-preserving spread, it is required that they be in $[0,1]$.

Thus we must have

$$(2.21) \quad \frac{2v_1 + v_2}{3} < \frac{v_1 + v_3}{2} < \frac{2v_3 + v_2}{3}$$

or,

$$(2.22) \quad \frac{2v_3 + v_2}{3} < \frac{v_1 + v_3}{2} < \frac{2v_1 + v_2}{3}$$

and,

$$(2.23) \quad \frac{2v_1 + v_2}{3} < v_2 < \frac{2v_3 + v_2}{3}$$

or,

$$(2.24) \quad \frac{2v_3 + v_2}{3} < v_2 < \frac{2v_1 + v_2}{3}$$

Without loss of generality, let us consider the case of $v_1 < v_3$, and hence of (2.21) and (2.23) - the case of (2.22) and (2.24) will be entirely symmetric.

From (2.23) we have

$$(2.25) \quad v_1 < v_2 < v_3$$

and from (2.21) we have

$$(2.26) \quad v_1 < 3v_3 - 2v_2$$

and

$$(2.27) \quad v_3 > 3v_1 - 2v_2$$

Note that (2.25) implies both (2.26) and (2.27). Therefore $\beta \in [0,1]$ implies $\alpha \in [0,1]$ and the general necessary and sufficient condition for this is that

$$(2.28) \quad v_1 < v_2 < v_3$$

or

$$(2.29) \quad v_3 < v_2 < v_1 .$$

3. The General Problem

In this section we present a general framework in which this problem can be studied. For ease of notation and exposition we consider the finite case.

Let

$$S = \{1, \dots, n\}$$

denote the set of states of nature, whose typical element is written j . The space of observations is the set

$$X = \{1, \dots, m\}$$

whose typical element is written, i .

Information is formalized as a matrix

$$B = (b_{ij}) \begin{matrix} i = 1, \dots, m \\ j = 1, \dots, n \end{matrix}$$

where b_{ij} is the probability of state j conditional on the signal being i , and a vector

$$\xi = (\xi_1, \dots, \xi_m)$$

where ξ_i is the probability that signal i will be observed.

The distribution of the states of nature is

$$\sigma = (\sigma_1, \dots, \sigma_n) .$$

The information structure must therefore satisfy

$$(3.1) \quad \sigma = \xi B .$$

Below, we will be concerned exclusively with information structures such that (3.1) holds.

We want to have conditions jointly on the information structure and on the underlying economic model that will enable us to decide whether the information structure will be valuable to all agents. In the present context, the economic model is described by a pair of vectors.

$$v = (v_1, \dots, v_n)$$

$$p = (p_1, \dots, p_n)$$

where (v_j, p_j) represents the revenue and price associated with the j^{th} state of nature.

The conditional mean prices are just the elements of the vector Bp and the overall mean price is ξBp .

We wish to compare the two decision problems, with and without information, as stated below. In each case, the choice to be made is the level of purchases of futures contracts, denoted by z if unconditional or by (z_1, \dots, z_m) if conditional on the observations.

Without information:

$$(I) \quad \max_z \quad \sum_j \sigma_j \quad u(v_j + z(p_j - \xi Bp))$$

With information:

$$(II) \quad \max_{z_1, \dots, z_m} \quad \sum_i \xi_i \quad \sum_j b_{ij} \quad u(v_j + z_i(p_j - (Bp)_i))$$

For each utility function, u , the value of I depends on σ , v and p ; the value of II depends on the information structure ξ and B as well.

Let us hold σ and p fixed, and write these values as

$$\phi^I(u, v)$$

and

$$\phi^{II}(u, v, \xi, B)$$

for information structures satisfying (3.1). We are interested in classifying the set of all triples

$$(v, \xi, B)$$

such that

$$(3.2) \quad \phi^I(u, v) \geq \phi^{II}(u, v, \xi, B)$$

holds for all concave functions u , or such that the above inequality is reversed for all concave u .

To approach this problem it is natural to define,

$$\mathcal{J}^+(v) = \{ (\xi, B) \mid (u, v, \xi, B) \text{ satisfies (3.2) for all concave } u \}$$

$$\mathcal{V}^+(\xi, B) = \{ v \mid (u, v, \xi, B) \text{ satisfies (3.2) for all concave } u \}$$

and $\mathcal{J}^-(v)$ and $\mathcal{V}^-(\xi, B)$, similarly, with the inequality in (3.2) reversed.

Some simple properties of these sets are evident and we list them below:

Property I

If v satisfies $v_j = \alpha p_j + \alpha_0$, for each j , then all information structures are in $\mathcal{J}^+(v)$.

Property 2

For any (ξ, B) , the set of all v such that $v_j = \alpha p_j + \alpha_0$ for each j is in $\mathcal{U}^+(\xi, B)$.

Property 3

For each (ξ, B) , $\mathcal{U}^+(\xi, B)$ is a convex set of vectors.

Property 4

For each v , the "no-information" structure in which all the rows of B are identically σ (and ξ can be arbitrary) is in $\mathcal{J}^+(v)$.

but

Property 5

$\mathcal{J}^+(v)$ is generally a non-convex set.

It is the last property that causes the difficulty in solving this problem. This is intimately related to the results of Radner and Stiglitz (1975).

They study the question of the value of improving the information structure slightly, beginning in a "no-information" situation. Under appropriate differentiability assumptions it is shown that the value of information is convex at the "no-information" point, and that the marginal value of information at this point is zero.

It is well-known that with a constant information structure, the introduction of variations in prices that maintains the same mean price will be harmful. The utility attainable in the system with fluctuating prices will be concave in the degree of fluctuation, at the "no-information" point.

The utility attainable in the present model can be written as the sum of these two effects. The marginal value of information at "no-information" is zero, and an analysis of the conflicting higher order terms is required to ascertain the value of even small changes.

To see this we consider a family of information structures parameterized by λ , denoted

$$\{B(\lambda), \xi(\lambda)\}$$

with the property that at $\lambda = 0$, the rows of B are identical and are all equal to σ . This value of the parameter represents the "no information" point.

Writing out problem II, above at the information structure $(B(0), \xi(0))$ we find that

$$\phi^{II}(u, v, B(0), \xi(0)) = \sum_i \xi_i(0) \sum_j b_{ij}(0) u(v_j + z_i(0)(p_j - (B(0) \cdot p)_i)) \quad (3.3)$$

where $z_i(\lambda)$ is the optimal purchase of the futures contract conditional on signal i , given that the information structure is known to be $\{B(\lambda), \xi(\lambda)\}$.

Assuming differentiability of $u(\cdot)$, the functions $z_i(\lambda)$ satisfy

$$0 = \sum_j b_{ij}(\lambda) u'(v_j + z_i(\lambda)(p_j - (B(\lambda) \cdot p)_i)) \cdot (p_j - (B(\lambda) \cdot p)_i) \quad (3.4)$$

for each i .

Let

$$u_{ij}(\lambda) \equiv u(v_j + z_i(\lambda) \cdot (p_j - (B(\lambda) \cdot p)_i)) \quad (3.5)$$

Differentiating (3.3) at $\lambda = 0$ we obtain

$$\begin{aligned} \frac{d\phi^{II}(u, v, B(0), \xi(0))}{d\lambda} &= \sum_i \xi_i(0) \sum_j \frac{db_{ij}}{d\lambda} u_{ij}(0) \\ &+ \sum_i \xi_i(0) \sum_j b_{ij}(0) \cdot (p_j - (B(0) \cdot p)_i) u'_{ij}(0) \frac{dz_i}{d\lambda} \\ &- \sum_i \xi_i(0) \sum_j b_{ij}(0) \cdot z_i(0) u'_{ij}(0) \sum_k p_k \frac{db_{ik}}{d\lambda} \\ &+ \sum_i \frac{d\xi_i}{d\lambda} \sum_j b_{ij}(0) u_{ij}(0) . \end{aligned} \quad (3.6)$$

where $u'_{ij}(0)$ is the derivative of the attained utility in signal i , state j with respect to income in that event.

Noting that $u_{ij}(0)$, $z_i(0)$ and $b_{ij}(0)$ are in fact all independent of i , we rewrite them as $u_j(0)$, $z(0)$ and $b_j(0)$ and can express (3.6) as

$$\begin{aligned} \frac{d\phi^{II}(u, v, B(0), \xi(0))}{d\lambda} = & \sum_j u_j(0) \sum_i (\xi_i(0) \frac{db_{ij}}{d\lambda} + \frac{d\xi_i}{d\lambda} b_j(0)) \\ & + \sum_j b_j(0) u'_j(0) (p_j - \sum_k b_k(0) p_k) \sum_i \xi_i(0) \frac{dz_i}{d\lambda} \\ & - z(0) \sum_j b_j(0) u'_j(0) \sum_i \xi_i(0) \sum_k \frac{db_{ik}}{d\lambda} p_k \end{aligned} \quad (3.7)$$

Each of the three terms on the right-hand side can be shown to be zero as follows. From the relations

$$\xi(\lambda) B(\lambda) = \sigma \quad \text{for all } \lambda, \quad (3.8)$$

we have

$$\sum_i \xi_i(\lambda) \frac{db_{ij}}{d\lambda} + \frac{d\xi_i}{d\lambda} b_j(\lambda) = 0, \text{ for all } \lambda \quad (3.9)$$

and hence the first term of (3.7) is zero. The second term is zero because of the first-order condition (3.4). Finally, using (3.9), the last term can be transformed into

$$z(0) \sum_j b_j(0) u'_j(0) \sum_k p_k b_k(0) \sum_i \frac{d\xi_i}{d\lambda} \quad (3.10)$$

Because

$$\sum_i \xi_i(\lambda) = 1 \quad \text{for all } \lambda \quad (3.11)$$

this is zero due to the term $\sum_i \frac{d\xi_i}{d\lambda}$.

Considering the value of information as it depends on the parameter λ , we see that it has a zero derivative at $\lambda = 0$, and may be either higher or lower at some intermediate value, as discussed in section 2. We seek general conditions, paralleling the second example of that section, under which utility would always be highest at the "no information" point.

One possible approach to this problem is the following:

Let us consider the payoffs attainable by taking actions z_i , $i = 1, \dots, m$, with information, and their associated probabilities.

$$(v_j + z_i (p_j - (Bp)_i), \xi_i b_{ij}) \quad \begin{matrix} i = 1, \dots, m \\ j = 1, \dots, n \end{matrix}$$

We want to know whether this random variable can be dominated by a suitable choice of z without information. This would attain the random variable

$$(v_j + z (p_j - \sigma p), \sigma_j) \quad j = 1, \dots, n$$

We know by virtue of Rothschild-Stiglitz (1971) that this will be true if and only if a set of weights $\{\alpha_{ij}^k\}$, $k = 1, \dots, n$; $i = 1, \dots, m$; $j = 1, \dots, n$ can be found such that

- (i) $\sum_{i,j} \alpha_{ij}^k = 1$ for each k
- (ii) $\alpha_{ij}^k \geq 0$ for all i, j, k .
- (iii) $\sum_{i,j} [v_j + z_i (p_j - (Bp)_i)] \alpha_{ij}^k = v_k + z (p_k - \sigma p)$ for each k
- (iv) $\xi_i b_{ij} = \sum \alpha_{ij}^k \alpha_k$ for each i, j .

For any information structure, one can view this as a system of inequalities in the v_j 's and z . It is therefore extremely difficult to describe $\mathcal{V}^+(\xi, B)$ because each z must be checked separately. Sufficient conditions for the information to be harmful can be derived if a particular candidate for z is chosen and the above system is solved for a set of conditions on the v_j . A likely choice is $z = \sum_i \xi_i z_i$, which in fact is the z that would induce the necessary and sufficient condition that the v 's be monotone in the p 's for the example of section 2. But more complex examples can be demonstrated in which the set of v 's in $\mathcal{V}^+(\xi, B)$ cannot be generated in this way. For any particular information structure the method of Fourier elimination (see Stoer and Witzgall (1970)) can be used to calculate the necessary and sufficient conditions for v to solve the system above.

The characterization of such relations is at present an open problem, to the best of my knowledge.

4. Conclusion

The primary goal of this paper has been to introduce the potential value of information through its aid in improving traders' hedging possibilities. This has been studied in a simple example, where it is shown that this beneficial effect may outweigh the adverse effects of induced price fluctuations. However, we have failed to provide a general characterization of the cases in which this dominance will occur.

Obtaining such a result would be important for practical considerations as well. Recent studies of the value of information in agricultural markets by Hayami and Peterson (1972) and Bradford and Kelejian (1975) have ascertained the short-run distributional consequences for improvements in information. Their models have two periods models and the traders cannot hedge risks by using futures markets; inventories held are the only decision variable. Because of the importance of such markets to agricultural decision making, a long-run analysis of the effects of changing information should rely heavily on the phenomenon described in this paper.

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