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Differential Information, the Market and Incentive Compatibility

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1 INTRODUCTION

In this survey of the economics of incomplete information, I will try to compare and contrast some of the more prominent models in this area by re-expressing them in the form of games of incomplete information. The explosion of research in this area over the past ten years makes a complete survey impossible. My goal will be to show that the theory of games of incomplete information, which has also experienced a renaissance of late, is the natural theoretical structure within which to classify and analyze more particular, specialized, economic models.

On the whole, however, the recent literature on the economics of information stands as a remarkable achievement in economic theory. It has demonstrated that useful, empirically testable conclusions can be derived in complex situations previously beyond analytical tractability. It has raised new theoretical questions and has solved some of the more puzzling older ones. It has given us a framework within which welfare analysis can be carried out. Though its interaction with game theory proper is still in its infancy, there is already reason to be hopeful about the fruits of this union.

There are many recently studied economic models that fall within the scope of this survey. Examples include the theory of auctions, from both a descriptive and normative viewpoint; the theory of labor contracts between workers and their employers; bilateral contracts; nonlinear pricing by a monopolist; adverse selection models in insurance; labor market signalling and mechanisms for preference elicitation. And there are many more. The common characteristic in all these theories is the absence of complete information on the part of point for present purposes is that each strategy n-tuple gives rise to a unique feasible outcome. All the players know the function g, and the evaluations of all the others. They also know that each of the others knows this, etc. In such a state of affairs the term 'common knowledge' has been used to describe the mutual awareness about the game and the other players.

Game-theoretic models are characterized by the idea that each player's strategy is chosen from a fixed set of possibilities and that the entire description of economic activity can be broken down into the actions of economic agents. This seems self-evident, as every action is the action of someone. However, it does not leave a role open for the economist's favorite construct – endogenous variables controlled by the 'invisible hand.' These variables may enter agents' objective functions, modify the constraints that they face, or alter the attainable set of allocations for the economy as a whole. If the endogenous variables are not set at their equilibrium values, the choices of individual agents will not be consistent.

Gerard Debreu (1952) was the first to take a model including endogenous variables and recast it in the form of a 'game.' His idea is elegant, simple and natural. One introduces a 'zeroth' (0th) player whose strategy is the selection of these endogenous variables. This agent's objective is to avoid infeasible collective actions, that is to insure that individual actions result in a social state $x \in X$.

To accomplish this at a formal level, it was necessary to extend the notion of a game in two ways. First, the space of outcomes was expanded to include those that could be contemplated and evaluated but which may not be feasible given actual economic resources. Second, instead of fixed strategy spaces, the strategy spaces of each player were allowed to depend on the strategy choices of others. In the competitive model of economic equilibrium, the price-setting actions of player zero affect each agent's choice set by changing the set of budgetarily feasible net trades available. The invisible hand was thus made visible. Games in this form are called *generalized games*; the 'generalization' at a formal level is in the mutual dependence of players' strategy spaces.

These two descriptions of economic models, as either games or generalized games, are quite condensed. They suppress all the detail about how the actual choices are made. These 'details' are of the essence in situations of incomplete information. Agents' decisions will be based partly on the information conveyed to them by other players, or by 'the market.' The nature of this information is endogenously determined in the equilibrium of the game or of the generalized game used to describe the economic model. In order to bring these considerations into the foreground, it is necessary to review the relationship between extensive and normal form games. This is the subject of the following section.

3 GAMES OF INCOMPLETE INFORMATION

Extensive and normal forms

The term 'games of incomplete information' is used to convey the idea that there may be some aspects of agents' environments about which they are uncertain, and that this uncertainty affects the conduct of their play. Games of incomplete information are games, in precisely the sense used in the last section: strategies collectively determine outcomes, and any strategy combinations are jointly feasible. The extensive form of games of incomplete information has been very important throughout the history of game theory. It is an attempt to describe the temporal structure of players' moves and of the evolution of their information. Precise mathematical formalizations are presented in several sources, of which Kuhn (1953) and Kreps and Wilson (1982) are perhaps the most readable.

The state of the game is associated with the nodes of a 'tree.' Associated with each non-terminal node is the name of a player whose turn it is to 'move.' To each terminal node one associates a social state, or a vector of payoffs to the players. Nature is also a player. When nature moves, the choice is made according to some given probabilities. Other players' moves can be random, but the choice of probabilities is at their own discretion.

When it is the turn of a player to move, he may not know the state of the game precisely. This is captured by partitioning the set of nodes at which a given player is to move into 'information sets.' The same choice, deterministic or stochastic, must be made throughout an information set. For this to make sense, the structure of the tree must be such that the same number of choices is available to a player throughout any given information set. At each terminal node of the tree the players have a payoff in von Neumann–Morgenstern utility.

In this very general situation, pure strategies are specifications of a choice of move at each information set. Mixed strategies allow for randomized choices. Each vector of mixed strategies determines a random path through the game tree and a distribution over the terminal nodes. The payoff to each player is just the mathematical expectation of his payoff at each of these nodes.

An *equilibrium* of a game is a situation in which each player's strategy is optimal given all the other players' strategies. For many purposes the equilibria of the game associated with the economic model characterize precisely the points of interest. If there are multiple equilibria, the theory does not predict which of them will 'actually' arise. The economic model does not provide a determinate theory, and economists have learned to live with this multiplicity of solutions.

In some other circumstances, however, there are equilibria of the associated game that seem not to describe a reasonable economic outcome. These cases arise quite frequently when the incompleteness of information is important. To eliminate these equilibria from consideration, much recent research has been devoted to imposing further restrictions on the equilibria. The essence of these restrictions is that players' strategies are required to be rational not only at the beginning of the game, but also at subsequent stages. This line of research has proven extremely valuable in narrowing the family of equilibria to usefully well-defined subsets that make economic sense. For most of this survey, however, we will see that the ordinary Nash equilibria of the games to be studied define the economic outcomes of interest. The refinements mentioned above will be of more importance in dynamic models with incomplete information, and this survey concerns primarily static situations.

The description of the extensive form of a game of incomplete information given above is incomplete if the economic model being studied involves endogenous variables that are not under the explicit control of any player. As mentioned in section 1, these cases correspond to generalized games, in which 'player 0' chooses the endogenous variables. To complete the description of the game we must provide player 0, somewhere during the course of play, with some of the information originally in the hands of the actual economic agents. To my knowledge, the only special case that has been examined is where player 0 is endowed with all of the information in the economy. It is then argued that, in an equilibrium of the generalized game, player 0's strategy will communicate all of this information to all of the participants.

This analysis of generalized games with incomplete information is incomplete in two respects. First, it does not give a realistic account of how player 0 comes to possess all this information. Second, the result about full dissemination is not robust to generalization with regard to the space in which the join of all private information lies. When this space is of higher dimensionality than the space of endogenous variables selected by player 0, serious problems concerning the existence of equilibria may arise. For these reasons, most of the rest of this survey will concentrate on games, rather than generalized games, of incomplete information.

Games with nature's move first

Any extensive game can be rearranged so that the moves of nature occur at the beginning. A move of nature occurring at a later stage can equally well be thought of as having been predetermined, but the information sets are such that its outcome is not yet known. Obviously, such rearrangements are strategically equivalent. We will adopt this format from this point onwards.

Many economic models have the feature that the only incompleteness of information suffered by the players concerns the choice made by nature. Although they may be differentially informed about nature's move, they have common and complete information about each other's moves. Typically, nature's move is interpreted as a parameter relevant to the preferences of all agents. One common specification of this idea is that agents' preferences are independently drawn from a fixed distribution, and that the preferences are purely individualistic in that they concern consumption of private goods without externalities. Another specification is that there is some common underlying parameter relevant to all their preferences and that nature's move consists of the choice of this parameter and the provision of an observation by each player that is statistically related to the parameter. In this model, the observations define 'conditional preferences.' As further information is exchanged during the course of play, the conditional preferences will evolve.

Other economic models allow for incompleteness of information by the players concerning each other's move, as well as about nature's move. Sometimes one of the players moves immediately after nature; his move is observable by others, and the remaining players then move simultaneously, in ignorance of each other's desires. These players can therefore condition their choices on the prior choices of the first player, as well as on any information they may have about nature's move.

A very important class of extensive-form games of incomplete information arises when all players must make a simultaneous choice, after nature's move is made. They are differentially informed about nature's move. Moreover, because the structure of the game is common knowledge, each can make inferences about what information is available to others. The equilibria of this type of extensive form are useful in analyzing the efficiency properties of outcomes. We consider this particular game in some detail in the next subsection.

The simultaneous-play extensive form

The simultaneous-play extensive form (SPE) is a family of games characterized by the facts that nature moves first and then players are informed of nature's move, although perhaps imperfectly. Then, simultaneously, the players make choices that, together with the true state, result in an outcome. There is no opportunity to revise actions or to communicate explicitly with the other players. SPE games differ according to the relationship between the space of possible choices by players and the space through which information about nature's move is conveyed to them.

In SPE games a strategy for player i is a mapping (possibly random) from the space of observations Y_i to the space of actions A_i. Let us denote these functions $a_i(y_i)$.

Following Harsanyi, we will say that $[a_i(y_i)]_{i=1,\dots,n}$ constitute a Bayesian–Nash equilibrium if they are an equilibrium of the game of incomplete information. The adjective 'Bayesian' refers to the fact that the choice of $a = a_i(y_i)$ is optimal for agent i given his beliefs about the unknown state and about the $(a_j)_{j\neq i}$ as generated by the $(y_j)_{j\neq i}$ and the other players' strategies $[a_j(\cdot)]_{j\neq i}$. These beliefs are generated by his prior knowledge and the observation y_i. The Bayesian–Nash equilibria of the SPE have been amenable to analysis in some detail, and have provided a useful benchmark in welfare theory, as we will see in section 4.

Of considerable practical importance is the further special case in which A_i and Y_i coincide – that is, have the same cardinality or dimension – for each player. Let A_i and Y_i be described by the same set. In these games there is a natural interpretation of any strategy. It describes the announcement that the player makes to all the others concerning his private information. The 'identity function' then corresponds to the ever-truthful strategy. A player's willingness to provide a truthful response depends on his posterior beliefs about others' observations, on their strategies, and on the outcomes associated to various professed observations. Much attention has been focused on discovering which outcome functions induce the truth as a possible equilibrium of this SPE. This is not because the truth is an objective *per se*, but rather because the allocation rules thereby achievable are an important family of outcomes of the game. This is discussed further in section 4.

Economic games without the SPE assumption

There are some games in which the SPE assumption is not appropriate. Only a few have been analyzed, as they are generally much more complex. The players can draw inferences from the actions of others and use them to modify their behavior at a later stage of the game.

One extensive form of this type is where the players pass through a sequence of simultaneous plays. At each stage they learn all of the moves made previously, but not any of the current moves of the other players. Some of these stages may correspond to 'communication moves' (Farrell, 1983).

The increased complexity of these strategy spaces makes a given game at least potentially capable of achieving superior equilibria. This is especially true when the set of possible moves at each stage is small compared with the space of observations (as is true in Farrell's case).

An interesting game of this form is the ordinary ascending, or English, auction. The price of an item is increased stepwise. Any player's move at a given price is either to 'stay in' or to 'drop out.' When only one remains in, he wins the item and pays an amount equal to the step at which his nearest competitors were active. Each player can observe how many of the others remain active at each state. Because of a correlation in valuations across individuals, each regards the number of active competitors he has as a relevant predictor of the value the item will have to him. An equilibrium of this game provides each player with a 'dropout function $k(p)$,' telling him to drop out at a price p if his own information is y and there are $k(p)$ or fewer other active bidders. This model has been used by Milgrom and Weber (1982b) to analyze how the seller's expected revenue responds to changes in the quality of information available and to the extent to which it is shared.

All of these games share the common feature that information about nature's move is not revised in the course of play, except in so far as it may be revealed by the strategies of other players. Whatever private information is available is given to the players at the beginning. It conditions their beliefs about the information, and hence the conduct of play, by the others. It is this aspect of the extensive form that makes these games much more tractable than the vast complexity that characterizes games of incomplete information in general.

4 WELFARE ANALYSIS, MECHANISMS AND GAMES OF INCOMPLETE INFORMATION

Welfare analysis has both positive and normative aspects. We may want to evaluate the outcome of a game against a standard of potentially achievable outcomes. Alternatively, we may examine the role of a social planner who must design a means of collective decision-making in order to achieve, in equilibrium, certain pre-specified objectives. In this section I will show how the theory of games of incomplete information is useful for both types of welfare analyses.

Following the notation of section 3, let $Y = Y_1 \times, \ldots, \times Y_N$ be the information revealed by nature to all of the participants, taken collectively. Because we assume that the revelation of nature's move is made exclusively at the first step in the game, before any actions of the players are taken, we can, without loss of generality, suppose that the set of feasible social outcomes depends on $y \in Y$. Thus, if X is the space of all conceivable social outcomes, the correspondence

$$\phi: Y \to X$$

describes the states that could actually be attained for some set of players' actions, given nature's move.

Each player i has preferences that depend on the pair $(y, x) \in Y \times X$. In any game or economic model, the *result* can be described as a probability distribution over $Y \times X$ having support on $\{(y, x) | x \in \phi(y)\}$. Let μ be a distribution over this set. Player i's evaluation of (y, x) is assumed to be expressed in the von Neumann–Morgenstern utility function

$$u_i: Y \times X \to R.$$

The *ex ante* expected utility of i is

$$u_i(\mu) = \int u_i(y, x)\, \mathrm{d}\mu.$$

After i learns his private information about nature's move he might evaluate the outcome μ somewhat differently. Letting $\mu(y_{-i}, x | y_i)$ be the conditional distribution of other players' private information y_{-i} and the outcome x given his information y_i, we can define, following Holmstrom and Myerson (1983), i's *interim* utility of μ given y_i as

$$U_i(y_i, \mu) = \int u_i(y_i, y_{-i}, x)\, d\mu(y_{-i}, x | y_i).$$

Similarly, the *ex post* utility of the result μ, when nature's move has been y, is the expected utility integrating over the conditional distribution of x given y:

$$U_i(y, \mu) = \int u_i(y, x)\, d\mu(x | y).$$

In alternative applications, each of the evaluations $U_i(\mu)$, $U_i(y_i, \mu)$ and $U_i(y, \mu)$ will be useful.

Let us use this formal structure to evaluate the performance of an economic system: the positive welfare analysis described at the beginning of this section.

Expressing the economic model as a game, each player has a strategy space, S_i, describing his play as it depends on his private information and on all his subsequent observations of other players' moves. The game is a mapping from the set of all possible strategies to the space of results. Let

$$g: \times S_i \to \Delta$$

where Δ is the set of all probability measures concentrated on $\{(y, x) | y \in \phi(x)\}$. Players choose their strategies $s_i \in S_i$ to maximize

$$U_i[y_i, g(s_i, s_{-i})].$$

An *equilibrium* $s^* = (s_1^*, \ldots, s_N^*)$ is a set of strategies such that each is optimal given the others.

The domain of positive welfare analysis is to discuss the efficiency of the equilibria of g relative to other strategy N-tuples in $\times S_i$. Note however that, although the game is played by each player maximizing his *interim* utility, it is possible to evaluate the results of the game according to the vector of *ex ante* utilities of the N players, or according to the vector of *ex post* utilities of the N players given $y \in Y$, as well as according to the interim utilities themselves.

Generally speaking, a game will not have efficient equilibria. The competitive equilibria of a neoclassical general equilibrium model are a notable exception. Extending this result to the case of incomplete information in general equilibrium theory has proved difficult – see Laffont (1983). Section 6 will be devoted to a summary of positive welfare results, recast in the general setting presented above.

Normative welfare analysis using this structure is also possible. Indeed, most of the results of the theory of incentive compatibility are of this type. There are two forms that this type of theory can take, according to whether or not there is an active mediator who can serve as a conduit for the information revealed to him.

In normative welfare analysis without a mediator, the strategy spaces S_i and the game g are considered variable. A planner or designer can choose $S = \times S_i$ and $g: S \to \Delta$ arbitrarily, subject to the restriction that $g(s)$ has support on $\{(y, x) | x \in \phi(y)\}$. We will refer to the planner's choice as a *design*. Once the design $(g, S_1, \ldots, S_N)$ is in place, its equilibria represent the results attained. The only active players in the design are the actual economic agents. The planner's only role is to set up the game.

One design can be compared with another on any of the three temporal bases described above. For example, an *ex ante* efficient design is one with an equilibrium result μ such that it is not possible to find another design having an equilibrium μ' and $U_i(\mu') \geqslant U_i(\mu)$ for all i, with strict inequality for some i.

A special form of design has been studied by Farrell (1983). These are called 'games of communication.' The strategies describe a two-stage process. In the first, the players make moves that are payoff-irrelevant in the sense that they have no binding force on their moves in the second part of the game, where strategic interaction actually determines the outcome. The purpose of the first part of the game is to make communication possible. The study of optimal designs is just beginning, but it clearly represents one of the important new ideas in welfare economics with incomplete information.

Normative welfare analysis with a mediator is a much more extensively explored research area. To model the set of attainable results when a mediator is present, we insert an active role for the mediator in the play of the game. The players' strategies are described in three stages. First, they are given a set of messages M_i that they can send to the mediator. Then the mediator sends a message $\hat{m}_i \in \hat{M}_i$ to the players, and finally the players choose their strategies in the payoff-relevant strategy space $\hat{S}_i$ as a function of their private information and the information revealed to them through the mediator's message.

Such a process is called a *mechanism*. It is described diagrammatically in figure 3.1. The mechanism is specified by the spaces $M_1, \ldots, M_N$, $\hat{M}_1, \ldots, \hat{M}_N, \hat{S}_1, \ldots, \hat{S}_N$ and the functions

$$h_i^1 : M_1 \times, \ldots, \times M_N \to \hat{M}_i$$

$$h_i^2 : \hat{S}_1 \times, \ldots, \times \hat{S}_N \to X.$$

The players' strategies f_i^1, f_i^2 are defined coordinate-wise:

$$f_i^1 : Y_i \to M_i$$

$$f_i^2 : Y_i \times \hat{M}_i \to \hat{S}_i.$$

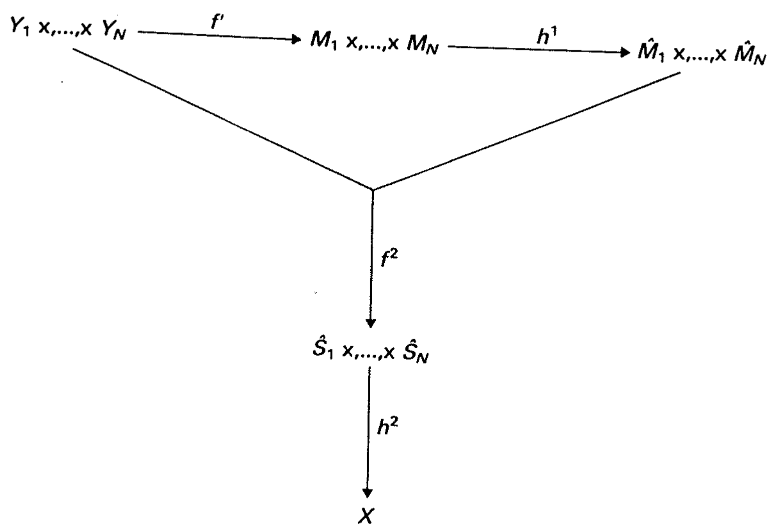

Figure 3.1

Mechanisms are more powerful than designs because the mediator can effect communication patterns via the choice of h^1 that could not be attained by decentralized agents. For example, player 1 could be informed of player 2's observation $y_2 \in Y_2$ only if his own observation were some particular $\bar{y}_1 \in Y_1$, and this observation could be hidden from him otherwise. Without a mediator this might not be possible – and in some cases it has been observed that it is socially desirable.

The structure of mechanisms presented above may seem to be very general and very complex. However, the basic results in the theory of incentive compatibility show that the mechanism design problem can be greatly simplified without loss of generality. A mechanism is said to *implement* a result μ if it has an equilibrium such that the induced joint distribution of (y, x) is μ. The *revelation principle* states that, if μ is implementable, we can find another mechanism in which $M_i = Y_i$ and $\hat{M}_i = \hat{S}_i$ for each i, and an equilibrium of this mechanism such that f_i^1 is the identity function for each i, and such that μ is also implemented by this mechanism. This reduces the mechanism design problem to the choice of $(\hat{S}_i, h_i^1, h_i^2)$. The objective, again, may be to optimize μ according to any of the criteria given above. Myerson (1983) has shown that this is a linear programming problem and has given a constructive method for its solution.

Notice that, if μ is implementable by a truth-inducing mechanism as described above, the game being played is actually of the SPE form discussed in section 3. Thus there is no loss of generality in restricting designs to the SPE form when a mediator is present. Without a mediator, the issue is more complex and remains unresolved at present. In particular, it is an open question as to whether the two-stage designs studied by Farrell can implement all the feasible results without a mediator, or whether some multi-stage strategy spaces, perhaps without simultaneous play, might be superior in some cases.

5 ECONOMIC MODELS WITH DIFFERENTIAL INFORMATION

I will now turn to a selective survey of models of economic behavior under conditions of differential information. The goal is to classify these models according to the special features embodied in each when formulated as a game of incomplete information. All of the systems studied are of the form described in section 3. Nature's move is differentially revealed to the participants at the beginning of play. The subsequent course of play, the stochastic structure of nature's move, and its revelation are the features that separate one system from another.

To describe the main lines along which I will make this classification, some preliminary remarks are necessary. Most of the models in the economics of differential information are partial equilibrium in nature. Something other than the agents' characteristics is held fixed, and is treated parametrically by the participants. There are two main types of 'partial equilibrium constraints.' First, there are conditions determining the voluntary participation or non-participation in the system. Alternative forms of these constraints are described and illustrated in the following subsection. Second, there are those auxiliary conditions related to endogenous variables determined in the system, other than the utility levels produced. Examples might be the prices faced by these agents or aspects of their market opportunities such as the competitive conditions in their industries. These obviously are an ingredient in determining their welfare, but they do not do so completely. The impact of these constraints is somewhat different in nature than explicit utility constraints. These models are discussed in the second subsection.

A third line of classification relates to the comparison of the strategy spaces of each agent with that agent's information set. In the SPE, discussed above in section 3, these sets were identified. Although this is of substantial normative significance, it turns out that in many actual games the strategy space is smaller than the space of observables. This feature leads to some problems in these models, as well as to some of their more interesting properties. It is covered in the third subsection.

Finally, we describe how the welfare properties of these models are influenced by their attributes in these regards.

One should note that the lines along which I distinguish these models do not form an exhaustive classificatory scheme. Some specific models have specific attributes mentioned in more than one of these subsections, and their properties are determined by the combined presence of these features rather than by any one of them alone. Some 'boxes' are 'empty.' Nevertheless, I hope that a discussion along these lines is useful.

Participation constraints

Consider an equilibrium of a game of incomplete information where $h: Y \to X$ is implemented. This produces the utility allocation $U: Y \to R$ defined by $y \nrightarrow u_i[h(y)|y]$ for each i. Let Φ be the set of all implementable utility allocations. Usually there will be many allocations in Φ, and often the planner, or mechanism designer, has a preference ordering over them. For example, X may include the possibility of making monetary transfers to the planner, for which the planner's utility is increasing. In such cases, the planner's problem of implementing that u which is best according to his criterion must be solved subject to some constraints. In the example above, the absence of constraints would permit the planner to increase uniformly the transfers to be paid by every y without inducing any distortions in the self-selection problem. Such large transfers are unreasonable.

The most natural type of constraints in these problems is 'participation constraints.' They express the ideas of voluntary participation in the process. Formally, there is an action available to each agent called *non-participation.* Any agent who chooses this action obtains a payoff $u_0(y)$.

The form of the participation constraints depends on the timing of the actions, the information-gathering and the commitment process inherent in the model. The natural points at which one can 'quit' are: *ex ante*, before learning y_i; interim, after learning y_i but before the interaction among the players produces a result; and *ex post*, after y is revealed and $u[y, h(y)]$ results. These lead to the three types of constraints:

$$\underset{y}{E}\, u_i[h(y), y] \geqslant \underset{y}{E}\, u_0(y)$$

$$\underset{y_{-i}|y_i}{E}\, u_i[h(y), y\,|y_i] \geqslant \underset{y_{-i}|y_i}{E}\, u_0(y)$$

$$u_i[h(y), y] \geqslant u_0(y).$$

Each of these forms of the participation constraint is appropriate for certain applications.

In procedures to select a public project, one should be able to obtain the *ex ante* consent of those involved that the process is beneficial. This is discussed extensively in Green and Laffont (1979). Given tastes, some people may have to pay more than the project is worth to them. But that is not a sufficient reason to reject the procedure.

On the other hand, when scarce private goods are involved, it is often more reasonable to allow the agents to select out of the procedures after they learn their valuation. Thus, auctions typically cannot collect revenues from those who concede all their chances of winning (see Milgrom and Weber, 1982a).

The difference between *ex post* and interim participation is of some interest when the private information does not precisely define the value of the object being auctioned. *Ex post* participation would allow the individual to withdraw his bid after learning its true value. This is not often reasonable, since the uncertainty about its value is one of the primary reasons for using the auction mode of allocation. Interim participation seems better in this case. *Ex post* participation is most relevant when one can voluntarily quit the organization – as in the case of local public goods in a small community.

Auction theory has usually used the interim constraint in a special way. The private information, Y_i, of each agent has been modeled as a real number, or an observation from an ordered set, with respect to which the utility function is monotonic. A lowest possible value of Y_i called $\underline{y}_i$ exists. Furthermore, it is assumed that the utility of withdrawal from the auction is independent of the true value of the item being sold. This is reasonable for items that are not subsequently resold to the same group of participants, but it may not hold in general. These two hypotheses imply that, if all agents are to be induced to participate, then it suffices to make the agent with $y_i = \underline{y}_i$ participate. The fact that the interim participation constraint can be expressed so simply provides auction models with a good deal of their tractability. The general nature of participation constraints in models with a one-parameter family of agents' preferences is explored in detail by Cooper (1982).

Auxiliary equilibrium conditions

By auxiliary equilibrium conditions I mean those constraints on the outcome of the model that are expressed as relations among endogenous variables other than the utilities or expected utilities produced in the equilibrium. Auxiliary equilibrium conditions are therefore intrinsically of a partial equilibrium nature. They have been most commonly used in economic models in which the uncertainty concerns individuals' unobservable characteristics and in which the statistical distribution of these characteristics in the population is effectively known because of the large numbers of individuals. This situation is not necessary for the presence and operation of auxiliary equilibrium conditions, but it makes their treatment much simpler.

Models of signalling (see Spence, 1973) are a clear example of the application of auxiliary economic conditions in a situation of incomplete information. Consider the prototypical example of unknown productivity signalled by educational attainment. The equilibrium is characterized by a function associating a wage to each level of education. This induces a joint distribution of productivity and education. In equilibrium it is also required that, for each level of education, the wage rate equals the average productivity conditional on that education. Thus the system is constrained not by any agent's utility, but rather by the form of the equilibrium relations.

It may be possible to convert the signalling model's constraint into a participation constraint of the type discussed earlier by treating the employers explicitly. They hire labor, paying the indicated wage, and in order to make them participate their profits (utility) must be non-negative.

I do not know whether it is always possible to integrate the utility constraints and auxiliary equilibrium conditions. It is possible above because of the large-numbers assumptions, making profits non-stochastic and identifying 'profits' with the 'utilities' of particular agents. In more general circumstances this may not be possible. In any case, making a distinction between these types of constraints seems analytically useful.

Dimensionality of observables and unobservables

Many of the features discussed in the last two subsections have special properties because of the equality of the dimensionality of the space of unobservables with the strategy spaces, and observable choices, of the economic agents. Indeed, all the models used to date have the property that the observables are one-dimensional and the unobservables are either one-dimensional or else consist of a discrete set of points – that is, a special type of zero-dimensional set. The advantage of this formulation is that the self-selection constraints, in the presence of a monotonicity condition (see Riley, 1979), can be expressed as a differential equation (in the former case) or as a system of inequalities with some very special properties (in the latter).

The more general problem of higher-dimensional spaces of unobservables remains almost totally unexplored. For a decision rule to be implementable, it is necessary both that it respect the incentive compatibility constraints and that, viewed as a function from the unobservables to the individual's outcome, it be of a rank not greater than the dimensionality of the pre-specified strategy space. J. Green and Laffont (1982) and E. Green (1982) have shown that this conjunction of these two conditions is extremely restrictive. One cannot specify a mapping from the unobservables to the strategies arbitrarily and then append a mapping from strategies to outcomes so as to make the indicated strategies of the game of incomplete information form an equilibrium. In general, there will be no game form, other than trivial cases where the social state is unresponsive, that induces the specified pattern of equilibrium strategies. The equivalence classes of unobserved parameters that can be induced by some game form are a predetermined family, related to the functional form of agents' objectives.

One case in which it is natural to construct a game with a smaller strategy space than the space of observables arises when the information is verifiable. The observer has, in his possession, some reproducible evidence that could be divulged to others. Moreover, the others know that, if he has in fact made certain observations, he will have the evidence to back them up. His choice therefore is to disclose the information or not. But he may not lie and say that he has seen some other observation, for he will not have the evidence to make it credible, public knowledge. Such a model is explored in Green and Laffont (1983).

6 WELFARE ANALYSIS OF ECONOMIC MODELS WITH DIFFERENTIAL INFORMATION

One advantage of having recast economic models with differential information as games of incomplete information is that we can perform a welfare analysis of their equilibria on a sound footing. To make a meaningful comparison of the equilibria of an economic model with those of a hypothetical alternative, it is essential to preserve both the information structure of the economic model and the individual's incentives. The comparison concerns the equilibria of the game of incomplete information induced by the economic model and the equilibria of a game designed by the economic analyst. This new game must use the same initial structure of information as the economic game. It must provide nature, or the artificial 'player 0,' with only such information as might properly be thought to comprise prior information. In particular, we cannot *give* this central player any information specific to economic agents – although the game may be structured to elicit this voluntarily from them.

Quite surprisingly, many of the economic systems discussed in section 5 are efficient in this sense. As this is certainly not true of games of incomplete information in general, we inquire in this section as to the reasons why efficiency arises in these cases.

The economic systems studied in section 5 can be divided into those that are actually mechanisms, or games among the economic agents, and those that require the equilibrating variables to be determined by a market manager who is not among these agents. The issue of efficiency is much simpler for the mechanisms. Here, interim efficiency is often established in the usual economic environments, with outcomes and private information individual-specific and without externalities, and with the aggregate feasible set non-stochastic. The reason is simply that interim and *ex post* efficiency coincide in these environments, and *ex post* efficiency is ensured because of the anonymity of the mechanisms considered and the monotonicity of *ex post* utility evaluations in the underlying parameters.

An example of these mechanisms is auctions in which all economic agents are on one side of the market and where their information is an estimate of the value of the good to them having the monotone likelihood ratio property. The highest evaluator will always win the auction. Note, however, that there are some situations where interim efficiency does not obtain. These are models in which there is random variation in the evaluations on both sides of the market, as in Weitzman's (1974) study of price versus quantity trading rules, Green and Honkapohja's (1983) more general model of long-term contracting, or Wilson's (1982) model of a one-shot sealed-bid double-auction (both sides bidding, and market-clearing taking place at a price determined by these bids).

These models are *ex post* inefficient because some initially beneficial trades are not undertaken in equilibrium. It follows, therefore, that they are interim and *ex ante* inefficient. Note that in Green and Honkapohja (1983) the incentive-compatibility-constrained efficient set is characterized subject to a dominant strategy equilibrium being achieved among the agents. The game they describe is implicitly not the SPE, where the solution concept is that Bayesian–Nash is played by each agent after he learns the true value of his parameter. Wilson uses the Bayesian–Nash solution concept. His analysis demonstrates that the double-auction is not an efficient mechanism, and that, when uncertainty affects both buyers and sellers, the efficient mechanism is likely to depend on the statistical distribution of the willingness to pay. This is to be contrasted with the one-sided auction models. There, the attainable utility frontiers for each valuation depend upon this distribution, but any one of several mechanisms (in the risk-neutral case) can always achieve a point on this frontier.

Finally, we come to the economic models that are not mechanisms and are not modeled as games in the SPE form. I will discuss two varieties: the learning-from-prices models of Green (1973) and Grossman (1981), and the signalling models of Spence (1973) and Riley (1979). The former do not result in interim-efficient allocations, whereas the latter have one interim-efficient equilibrium and many other inefficient equilibria.

Let the agent's observations in the learning-from-prices model be y_i and let prices be formed by the market-maker according to the function $p(y_1, \ldots, y_n)$. Agent i's indirect utility, *ex post*, can be written as $v_i(y_1, \ldots, y_n, p)$, which, for each fixed price vector, would represent the maximized value of his utility given all the information. Integrating out y_{-i} we obtain the indirect interim utility $v_i(y_i, p)$. Because p is still a random variable given y_i, the agent is at risk for the fluctuations in p induced by other agents' information. An interim-efficient mechanism would insure the agent against these risks. The learning-from-prices models permit some of the risks to be mitigated; but only when all information is effectively conveyed by prices, as in Grossman (1977) or Radner (1979), is full insurance possible. When prices do not form a sufficient statistic, as in Bray (1981), an alternative mechanism based on such a statistic would be able to interim-dominate the learning-from-prices equilibrium.

Signalling models do not have this difficulty. The price vector is non-stochastic because there is no aggregate uncertainty in the model. The incentive-compatibility constraints are effectively built into the signalling equilibrium wage structure. Therefore they do not allow any ability group to benefit without a corresponding decrease in utility for a higher ability group, or else a violation of the implementability requirement.

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