

Renegotiation and the Form of Efficient Contracts

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ABSTRACT. — Two parties may agree to a mutually binding contract that will govern their behavior after an uncertain event becomes known. As there is no agent who can both observe this uncertain outcome and enforce the contract, contingent agreements are precluded. However, the parties recognize that the uncertain event will be common knowledge for them, and that they will be able to renegotiate the contract voluntarily, provided that they both gain in doing so. When structuring the original contract they can foresee this renegotiation phase. Efficient contracts are those that perform best, when taking this into account.

This paper studies the form of such efficient contracts. It is shown that it is always better to have a contract than it is to have none, no matter which party has the preponderance of bargaining strength in the renegotiation phase. We also study whether renegotiation can substitute completely for the absence of contingent contracts. We characterize a family of cases where it can. And we present some "second-best" results in others, where it cannot.

Renégociation et forme des contrats efficaces

RÉSUMÉ. — Deux parties peuvent *ex ante* se lier par un contrat qui gouvernera leur relation une fois qu'un événement aléatoire se sera réalisé. En l'absence d'une tierce partie qui pourrait observer cet événement et contraindre les parties à réaliser des échanges, des contrats contingents à cet événement sont impossibles. Cependant, les parties peuvent reconnaître que l'événement aléatoire deviendra connaissance commune pour elles et qu'elles auront la possibilité alors de renégocier pourvu qu'elles y gagnent toutes les deux. En structurant le contrat original les parties peuvent anticiper ces renégociations. Des contrats efficaces sont les meilleurs contrats qui prennent en compte cette phase de renégociation.

Cet article étudie la forme que doivent prendre ces contrats efficaces. On montre d'abord qu'il est toujours préférable d'avoir un contrat plutôt que de ne pas en avoir quelles que soient les allocations de pouvoir dans la phase de négociation.

Ensuite nous étudions dans quelle mesure la renégociation peut être un substitut complet à l'absence de contrats contingents. Nous caractérisons une famille de cas où cela est possible et nous présentons quelques résultats de second rang lorsque ce n'est pas possible.

1 Introduction

In this paper we discuss the process of contract formation and contract revision in a two-party relationship. At the time that the contract can first be written some uncertainty about the future is present. However, although both of the contracting parties can observe the resolution of this uncertainty before any payoff-relevant actions are required, we assume that it is not possible to implement contracts which are specified *ex ante*, before the uncertainty is resolved, and are contingent upon these outcomes. This contractual "incompleteness" or "imperfection" is at the heart of the phenomena we study and is justified as follows. As parties have conflicting interests the contracts needs to be enforced by some third party, or outside agency, who has, or is invested with, the power to punish the participants for failing to comply with its provisions. A "full" specification of the contractual relations would include the actions to be taken in various contingencies and the enforcement procedures to be used. In our model, the outside enforcement agency cannot observe the resolution of uncertainty at all. This is a way of capturing the more realistic viewpoint of limited, costly, observability which more accurately characterizes reality. (Further discussion of the use of this non verifiability assumption is given below.)

Having stated what the enforcement agency cannot do, we must be explicit about the powers with which it is endowed. We assume that the enforcement agency can observe the chosen action and can compare it to the contractual specifications. These specifications can be a precise description of unique actions that have been agreed upon, or, more generally, can consist of a set of allowable actions within which one of the parties has the full discretion to choose the actual result. If the chosen action violates these specifications, a large (unspecified) punishment can be inflicted. For those contracts in which limited control is delegated to one side or the other, the action chosen within the allowable range will be that in the best interest of the controlling party.¹ Moreover, the agency can observe which party has been given the power of choosing the "renegotiated" contract.

This paper is concerned with the nature of optimal contracts when it is known that the contract will be renegotiated. The effect of renegotiation is to modify the original agreement. Thus the utilities that would have been achieved if the original agreement were enforced are the status quo

utilities in the renegotiation phase. We will assume that the nature of renegotiation is common knowledge and thus both practices can accurately predict how each contract will result in a final allocation of utilities as a function of the initially uncertain state.

The first principal focus of this paper is to ask whether a contract will be signed *ex ante* at all, or whether agreement will be delayed by not writing an initial contract. Not having any contract means that the utility of the agent at the renegotiation phase is the same status quo level that was available *ex ante*. A first period contract determines status quo levels which are in general a function of the unknown information. Without a contract, the status quo utility is a constant. Therefore the superiority of an *ex ante* contract, **followed by renegotiation**, over no contract followed by an *ex post* efficient agreement is not obvious. This is the first main result we establish.

The second principal focus of the paper is the renegotiation of contracts. When a fully optimal contract is not possible at the initial date, it is natural to presume that *ex post* inefficiencies will be eliminated by mutually advantageous renegotiation. Renegotiation thus results in a variable outcome and the parties are exposed to some risk. In a first best this risk will be allocated efficiently. We inquire when the first best can be reached through renegotiation. And if it cannot, then what is the second-best form of the initial contract?

Before outlining our results it may be useful to quickly review the literature. WILLIAMSON [1975] should be credited for emphasizing the problems that can develop in contracting when parties have *ex post* symmetric but non verifiable information.

"Both buyer and seller have identical information and assume, furthermore, that this information is entirely sufficient for the transaction to be completed. Such exchanges might nevertheless experience difficulty if, despite identical information, one agent makes representations that the true state of the world is different than both parties know it to be and if in addition it is costly for an outside arbiter to determine what the true state of the world is".

Since information is *ex post* non verifiable contracts contingent on this information cannot be signed. GROSSMAN and HART [1985] consider the extreme case where no contracts at all are allowed *ex ante* except for the allocation of control *ex post*. They show that this allocation of control affects the *ex ante* allocation of specific non verifiable investment which is usually inefficient. However, traders can in general contract on publicly observable variables. Hostages or cancellation fees (WILLIAMSON [1983]),

1. Contracts based on a more interactive type of control, rather than on delegated choice, are possible. To administer these contracts both parties must have messages or strategies that are observable by the enforcement agency. We do not consider this type of contract here. See our later paper GREEN and LAFFONT [1991], or AGHION, DEWATRIPONT and REY [1991]. The analysis in this paper presumes that actions, such as the choice from a

BOLTON [1986], GROSSMAN and HART [1987] can be introduced to improve the allocation of resources.²

Section 2 defines the model we use and shows that the contract in which the control of renegotiation is given to the agent is fully general given the observability of information. Section 3 characterizes the first best allocations. We show in section 4 that, under quite general circumstances we can predict that a contract will be signed *ex ante* because no contract is dominated by a specific well chosen contract that we identify. Section 5 studies the general second best optimization problem that the principal must solve and characterizes some situations in which the first best is implementable. Examples are discussed in section 6.

2 The Model

2.1. Contracts

The two players will be called the principal and the agent. They choose two decision variables, x and t . In some applications x will be a quantity of some economic good, perhaps produced by the principal and consumed by the agent, and t will be a monetary transfer from the agent to the principal. In other applications, however, their interpretation may be quite different.

The two players are uncertain about a random variable θ , whose distribution they believe to be F . This variable is payoff relevant only to the agent.

2. There is no clear limit to the complexity of the contracts which can be designed *ex ante* to deal with the ex post non verifiable information. One possibility is to introduce bounded rationality and hence, limited complexity (see HUBERMAN and KAHN [1986 a, b] for example). HART and MOORE [1985] specify a particular game form to represent renegotiation in which the court may or may not be able to observe messages sent between parties ex post. Their main focus is on the underinvestment effect in specific relationships when parties are risk neutral. They also present a special case with risk aversion in which the first best can be achieved through their contracting design (see also GROUPE [1984], TIROLE [1986]).

The view held by the implementation literature (MASKIN [1985], MOORE and REPULLO [1985]) is that a lot can be achieved, often the first best. However, the games used in this literature rely on collective penalties which require the availability of a perfectly benevolent third party. In this paper we assume that such a third party is not available and that any attempt to use games based on non public information such as messages

Thus the two players von Neumann-Morgenstern utility functions are

$$(1) \quad \begin{cases} U_P = f(x, t) \\ U_A = u(x, t, \theta) \end{cases}$$

We will always assume that $f_x > 0$, $f_x < 0$, $u_x > 0$, $u_t < 0$ and $u_\theta > 0$. This last condition means that higher values of θ are, at constant (x, t) always "good news". The other assumptions follow quite naturally in the interpretation given above, and in others discussed below. We will also assume the usual single crossing property, $\frac{\partial}{\partial \theta} \left(-\frac{u_x}{u_t} \right) > 0$.

In a world of costless contracting and perfect contract enforcement the players would agree on a contingent arrangement regarding both x and t as a function of θ . This paper concerns cases in which such a plan is not feasible, at least by means of a direct contract fixed before θ becomes known. Let us therefore be precise about the type of contracts that are feasible, and about the structure of the process that determines how the players can enforce and modify the contract after they both learn the value of θ .

We assume that if a contract is offered at all, it is selected by the principal in such a way that the agent will achieve an expected utility of \bar{w} , his reservation level of utility. One option the principal retains is to offer no contract at all to the agent. If this happens, the principal must offer the agent a proposal, after θ becomes known, that still achieves this reservation value of \bar{w} .

A contract at the initial point in time can have two types of provisions. The first specifies an outcome, or a set of outcomes, that are agreed upon. The enforcement agency, which we sometimes call a "court", can observe whether the actual outcome coincides with, or, more generally, lies in, this contractual stipulation. The court has the power to enforce this provision - perhaps by inflicting a large unspecified punishment if it is violated - unless the parties voluntarily agree to replace this contract with some other agreement. In this sense the agreed upon outcomes are binding upon both parties.

If this contractual provision specifies a set of outcomes, it is understood that the agent retains the right to choose among them and to insist upon the outcome that is best for himself, unless a mutually superior outcome arises in the renegotiation phase. Note that it would never be useful to specify a set of outcomes and give the principal the right to choose among them because, as the principal's utility is independent of θ , his choice would always be the same and the sub-optimal members of this set would be irrelevant.

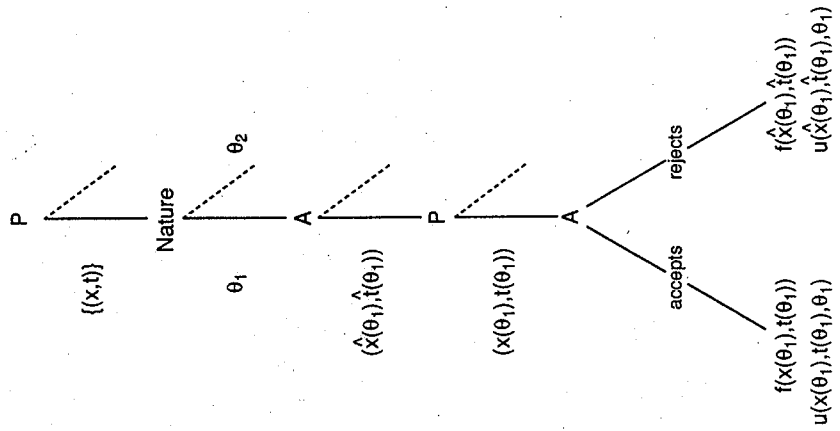
To summarize the first phase of a contract, there are two possible specifications. Either a set of outcomes (perhaps a singleton) is delineated and the agent is given control among them, or a single outcome is specified upon which the principal can insist.

circumstances. Although they may be understood perfectly well by the parties themselves, neither side may be able to actually prove to a court exactly what these circumstances are, or at least this may be infeasible in a reasonable amount of time and without much cost.

The second set of provisions of the contract relate to the time after θ is revealed, when the parties have the opportunity to renegotiate. The rules of the bargaining games that we consider are particularly simple. If the agent is given control over the possible outcomes at the first stage, the principal is given the power at this stage to make a take-it-or-leave-it offer that, if rejected, will result in the agent's original choice being enforced by the court. Conversely, if it is the principal who is guaranteed the right to insist upon a particular outcome, then the agent is given the exclusive right to hold the principal down to that level of utility in the renegotiation phase.

FIGURE 1

Delegation and P-led Renegotiation.

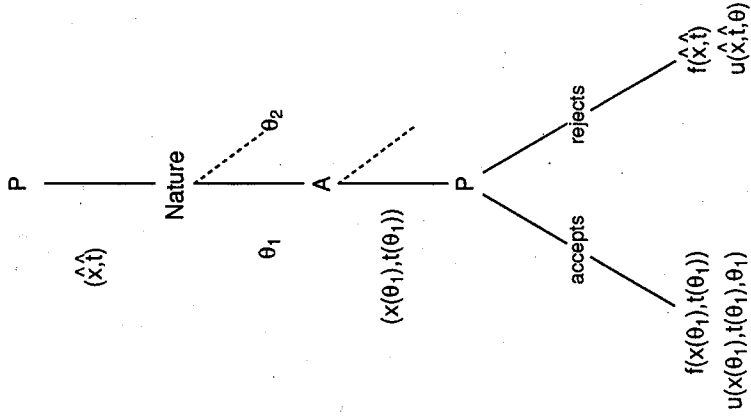


These two forms of control of the renegotiation phase exhaust the

well have included the allocation that this player would propose in the list of contractually specified possibilities and ignore renegotiation entirely.

FIGURE 2

A-led Renegotiation.



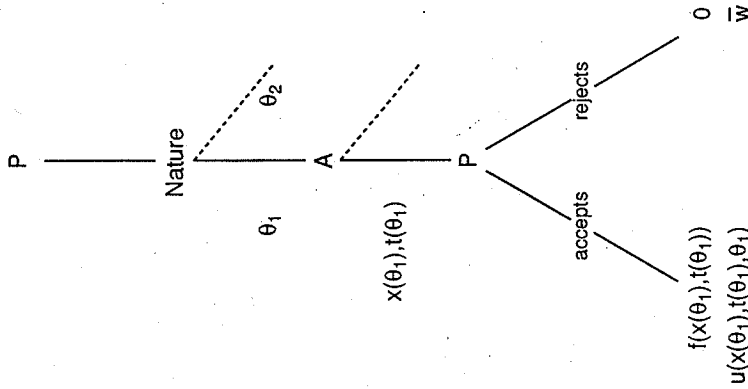
Thus there are two distinct forms of contract. The extensive form games corresponding to these contracts are shown in figures 1 and 2. The extensive form of the game corresponding to no contract is given in figure 3.

In either form of contract the court must be able to observe the outcome chosen by the party given control; it must be able to compare this outcome to the specified set of outcomes; and it must be able to preserve the right of the party specified as the Stackelberg leader in the renegotiation phase. If the court is given these powers and not others – for example the court cannot observe the strategies played in any normal or extensive form game the players might put in place by virtue of their contract – then these two forms of contract exhaust all the possibilities.

One might think that our specification of such a simple form of renegotiation – the making of a take-it-or-leave-it offer – is too simple and

FIGURE 3

No Contract.

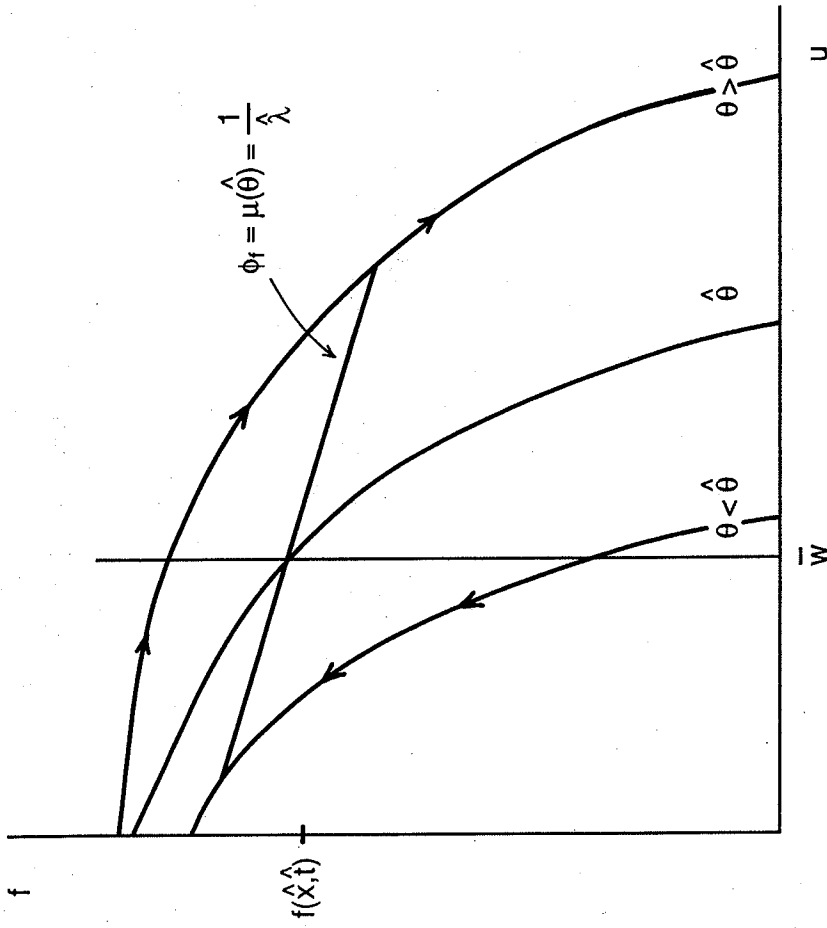


Nash solution with the original point being the disagreement outcome. This is, of course, possible.³ However any such renegotiation phase, if specified as an extensive form game, will necessarily involve strategic choices by both players. If these strategies are themselves unobservable by the court, there will be no way to insure that this other bargaining solution is, in fact, the result of such renegotiation. Thus, given our assumptions about what is and what is not observable by the court, the two contractual forms described above exhaust all possibilities.

2.2. Payoffs

Many of our results are obtained with the general payoff functions (1), possibly with the addition of some mild and well known qualitative

FIGURE 4



conditions. Two special models suggest themselves, and particular results that compare the efficiency of alternative contractual forms in these models will be given in section 5 and 6.

The first is a model of a buyer whose valuation function for a good he is purchasing is parametrized by θ . The buyer is the agent, x is the quantity purchased and t is the amount of money paid. If the buyer is risk-neutral in money, then his utility is

$$U_A = -t + u(x, \theta)$$

The seller, assumed to be risk averse, produces the good with constant returns to scale. Without loss of generality we can set unit cost at unity. Thus,

$$U_P = f(t - x)$$

The second model concerns a firm and a worker. The worker's utility for income, x , depends on θ . Perhaps the worker does not know the result

members, are unknown. Then, if the worker is risk-neutral in his effort, t , as above. The principal is the firm. The firm's profits are concave and increasing in the worker's effort. Hence,

$$U_A = -t + u(x, \theta)$$

as above. The principal is the firm. The firm's profits are concave and increasing in the worker's effort. Hence,

$$U_P = f(t) - x.$$

(Note that here the roles of t and x are reversed from that of the more usual notation).

2.3. General Results about the Efficiency of Alternative Contractual Forms

Let us introduce a somewhat abbreviated terminology for the contractual forms discussed in section 2.1. When the agent is delegated authority to choose the outcome within some set and the principal has the upper hand in the renegotiation phase, we will say the contract specifies *delegation and P-led renegotiation*. When the set in which the agent can select is reduced to a singleton, we say that delegation and P-led renegotiation reduces to a *simple contract* and P-led renegotiation. When the agent is given the power in the renegotiation phase we will say that the contractual form is *A-led renegotiation*. Finally, if no agreement is made *ex ante* we will call it *no contract*.

The results of any of these contractual forms will be a pair of realizations $x(\theta)$, $t(\theta)$, as determined by the rules of these games and the self-interest of the players. We will say, for example, that $x(\theta)$, $t(\theta)$ is *implementable via delegation and P-led renegotiation* if there is a contract of this form that results in these outcomes. Likewise for implementability via the other forms of contract.

THEOREM 1: The utility reached by the principal in a contract with A-led renegotiation can be reached or exceeded for all θ in a contract with delegation and P-led renegotiation.

Proof: In the proof, we first show that if $x(\theta)$, $t(\theta)$ is implementable via A-led renegotiation then it is also implementable in the ordinary sense of incentive compatibility (as if θ were the private information of the agent even though in fact it is not). Then we show that if $\hat{x}(\theta)$, $\hat{t}(\theta)$ is incentive compatible, then there will exist another pair $x(\theta)$, $t(\theta)$ that is implementable via delegation and P-led renegotiation that is preferred by the principal for all θ . The arguments are straightforward. Take $x(\theta)$, $t(\theta)$ implementable via A-led renegotiation. Then there exists \hat{x} , \hat{t} such that,

$$(2) \quad f(\hat{t}, \hat{x}) = f(t(\theta), x(\theta)) \quad \text{for any } \theta$$

and

for all (x, t) such that $f(t, x) \geq f(\hat{t}, \hat{x})$ in particular for all $(t(\theta'), x(\theta'))$, $\theta' \in \Theta$. This is precisely the condition for the incentive compatibility of $x(\theta)$, $t(\theta)$.

Now consider any incentive-compatible $x(\theta)$, $t(\theta)$. This means that for all θ, θ' ,

$$u(x(\theta), t(\theta), \theta) \geq u(x(\theta'), t(\theta'), \theta).$$

Delegate to the agent the right to choose within

$$\{(x, t) \mid x = x(\theta), t = t(\theta), \text{ for some } \theta\}.$$

Following P-led renegotiation the agent will achieve the utility $u(x(\theta), t(\theta), \theta)$. The proposal of the principal will generally be preferred to $(x(\theta), t(\theta))$. \square

Three remarks about this theorem should be made. First, the absence of θ from the utility function of the principal is crucial. Second, we cannot, *a priori*, rank the efficiency of "no contract" — this is the subject of section 4. Third, simple contracts with P-led renegotiation are not flexible enough to reach the entire incentive compatible family. Therefore, if for some reason the principal cannot delegate discretion to the agent, he may in fact be better off giving the agent the power to lead the renegotiation phase, despite the general result of this theorem to the contrary — see section 6.

3 The First Best

Let us begin by examining the first-best solutions to the problem, and the applications given in section 2.

The general problem, *ex ante* is

$$\max \int U_P dF(\theta)$$

subject to

$$\int U_A dF(\theta) \geq \bar{w}$$

The first-order conditions for this problem are

$$f_x(t^*(\theta), x^*(\theta)) + \lambda^* u_x(x^*(\theta), t^*(\theta), \theta) = 0$$

$$f_t(t^*(\theta), x^*(\theta)) + \lambda^* u_t(x^*(\theta), t^*(\theta), \theta) = 0$$

In the case of a buyer with unknown valuation facing a risk averse seller with constant unit costs, we have, in addition to (4),

$$f'(t^*(\theta) - x^*(\theta)) = \lambda^*$$

$$f'(t^*(\theta) - x^*(\theta)) = \lambda^* u_x(x^*(\theta), \theta)$$

Hence, eliminating λ^* , $u_x(x^*(\theta), \theta) = 1$.

In the case of a worker facing a firm with unknown productivity, we have

$$-1 + \lambda^* u_x(x^*(\theta), \theta) = 0$$

$$f'(t^*(\theta)) - 1 = 0$$

Note that in each case the agent's utility at the first-best is monotone in θ . This follows from the property that the cross-derivative $u_{x,\theta}$ is positive. In the case of the buyer and seller,

$$\frac{dU_A(x^*(\theta), t^*(\theta), \theta)}{d\theta} = -\frac{dt^*(\theta)}{d\theta} + u_x(x^*(\theta), \theta) \frac{dx^*(\theta)}{d\theta} + u_\theta(x^*(\theta), \theta) > 0$$

In the case of the worker and the firm,

$$\frac{dU_A(x^*(\theta), t^*(\theta), \theta)}{d\theta} = u_x(x^*(\theta), \theta) \frac{dx^*(\theta)}{d\theta} + u_\theta(x^*(\theta), \theta)$$

which is likewise positive because $\frac{dx^*}{d\theta} = -\frac{u_{x,\theta}}{u_{xx}} > 0$.

4 The Domination of "No Contract"

In this section we show that having "no contract" is, under quite general conditions, dominated by a contract in which the agent is given the leadership power in the renegotiation phase. By virtue of theorem 1, therefore, we will have shown that "no contract" can never be the efficient form of contract. The optimal contract will, in general, be of the form of delegation and P-led renegotiation.

The control of the relationship in the *ex post* bargaining phase, in practice, depends upon details of their specific irreversible investments and their foregone outside opportunities. There may be cases where the principal must give the agent the bargaining advantage in the renegotiation phase. The importance of the result in this section is that, in such cases,

One instance of this loss of bargaining power may arise when the principal is a worker contracting with a firm and the worker's unknown ability enters the firm's production function. Once the worker has worked for this firm for a while, long enough for his "ability", which may be the realization of his "long run potential", to become known, he may have lost his mobility or marketability in other jobs and may, therefore, be held to his contractual level of utility by the agent, his employer. According to the theorem of this section, it will always be more efficient, *ex ante*, for this worker to contract with the firm, rather than to wait until the uncertainty is resolved.

This result depends on some mild assumptions about the form of payoff functions. We first assume that both goods are "normal" for both players. That is the locus of pairs (x, t) at which the marginal rates of substitution are constant is monotonic in the x, t -plane. This condition can be expressed as,

$$\frac{d}{dt} \left(\frac{-u_t}{u_x} \right) > 0$$

$$\frac{d}{dx} \left(\frac{-u_t}{u_x} \right) > 0$$

$$\frac{d}{dt} \left(\frac{-f_t}{f_x} \right) < 0$$

$$\frac{d}{dx} \left(\frac{-f_t}{f_x} \right) < 0$$

These imply,

$$(5) \quad u_t u_{tx} - u_x u_{tt} > 0$$

$$(6) \quad u_t u_{xx} - u_x u_{tx} > 0$$

$$(7) \quad f_t f_{tx} - f_x f_{tt} < 0$$

$$(8) \quad f_t f_{xx} - f_x f_{tx} < 0$$

The principal result of this section can now be stated.

THEOREM 2: Assuming (5)-(8), and appropriate boundary behavior for the derivatives of f and u there exists a contract with A-led renegotiation which is superior for the principal to not offering any contract to the agent.

Proof: The proof is somewhat technical, though straightforward, and is given in the Appendix. The intuitive idea behind this theorem is the following: With no contract, the agent is held to a fixed utility level and thus the principal bears all the risk. Under a contract specifying (\hat{x}, \hat{t}) the agent's utility will vary monotonically with θ and hence the agent will bear some of the risk that otherwise would have fallen on the principal. Of

5 Implementation of First-Best Allocations

First, we can observe that only the mechanisms of delegation and P-led renegotiation have the potential of reaching the first best in generic situations. The no contract mechanism produces a constant (in θ) utility level for the agent, the A-led renegotiation produces a constant utility level for the principal; both are unlikely to characterize the first best. The incentive compatible allocations corresponding to delegation without renegotiation require a very particular relationship between $x(\theta)$ and $t(\theta)$ namely

$$x'(\theta)u_x(x(\theta), t(\theta), \theta) + t'(\theta)u_t(x(\theta), t(\theta), \theta) = 0 \text{ almost everywhere}$$

which will not be satisfied in general.

Therefore let us consider delegation with P-led renegotiation and consider $w^*(\theta)$ a level of utility achieved by the agent in a first best allocation.

We will say that $w^*(\cdot)$ is *implementable via delegation and P-led renegotiation* if there exists an incentive compatible mechanism $(x(\theta), t(\theta)) : \Theta \rightarrow \mathbb{R}^2$ such that:

$$(9) \quad u(x(\theta), t(\theta), \theta) = w^*(\theta), \quad \theta \in \Theta.$$

The principal then proposes this mechanism which provides the status quo levels $w^*(\theta)$ for the agent and ex post maximizes his objective function under the agent's individual rationality constraint:

$$u(x, t, \theta) \geq w^*(\theta)$$

reaching in this way an efficient allocation in which the agent obtains $w^*(\theta)$.

Incentive compatibility requires:

$$(10) \quad x'(\theta)u_x(x(\theta), t(\theta), \theta) + t'(\theta)u_t(x(\theta), t(\theta), \theta) = 0 \text{ almost everywhere}$$

Under the single crossing property (SCP), $\frac{\partial}{\partial \theta} \left(-\frac{u_x}{u_t} \right) > 0$, $\forall x, t, \theta$, we know (GUESNERIE and LAFFONT [1984]) that the second order condition:

$$(11) \quad x'(\theta) \geq 0$$

together with (10) yield necessary and sufficient conditions for incentive compatibility.

Therefore, we have the desired result if the solution $x(\theta)$ obtained from (9) (10) satisfies (11).

Differentiating (9) and using (10) gives:

Differentiating again (12) we have:

$$u_{\theta\theta}(x(\theta), t(\theta), \theta) - x'(\theta) \cdot u_t(x(\theta), t(\theta), \theta) - \frac{\partial}{\partial \theta} \left(-\frac{u_x(x(\theta), t(\theta), \theta)}{u_t(x(\theta), t(\theta), \theta)} \right) = w^{**}(\theta)$$

or in view of (SCP) and $u_t < 0$,

$$(13) \quad \text{sign } x'(\theta) = \text{sign } [w^{**}(\theta) - u_{\theta\theta}(x(\theta), t(\theta), \theta)].$$

From (13) we have immediately:

THEOREM 3: Under the single crossing property and $u_{\theta\theta} = 0$, the first best agent's utility profile $w^*(\cdot)$ is implementable via delegation and P-led renegotiation if $w^{**}(\theta) \geq 0$ and $w^{**}(\theta) \geq 0$ for any θ .

We will restrict now the analysis to the utility functions

$$(14) \quad -t + \theta v(x), \quad v' > 0 \quad v'' < 0$$

which satisfy the assumptions of Theorem 3.

In more general cases the argument above does not enable us to develop a necessary and sufficient condition because $u_{\theta\theta}$ is a function of the unknown mechanism $(x(\cdot), t(\cdot))$.

Let us call $\varphi(w, \theta)$ the solution of

$$\text{Max}_f f(t, x)$$

$$u(x, t, \theta) = w$$

i.e. the principal's utility level after renegotiation if the status quo level is w for the agent.

Under the assumptions of Theorem 3, and assuming $\Theta \equiv [\underline{\theta}, \bar{\theta}]$, the optimization program of the principal can be written.

$$\text{Max}_{w(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} \varphi(w(\theta), \theta) dF(\theta)$$

subject to

$$\int_{\underline{\theta}}^{\bar{\theta}} w(\theta) dF(\theta) = \bar{w}$$

$$w'(\theta) \geq 0 \text{ and } w''(\theta) \geq 0$$

Defining two state variables $w(\theta)$ and $z(\theta) = w'(\theta)$ and the control $\mu(\theta)$ we can rewrite this program:

$$\text{Max}_{\mu(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} \varphi(w(\theta), \theta) dF(\theta)$$

$$\int_{\underline{\theta}}^{\bar{\theta}} w(\theta) dF(\theta) = \bar{w} \quad (\lambda)$$

$$w(\theta) = z(\theta) \quad (v_1(\theta))$$

$$z(\theta) = u(\theta) \quad (v_2(\theta))$$

(16)

$$z(\theta) \geq 0$$

The shape of the optimal solution will depend on which of the constraints (15) or (16) is binding. If none of the constraints is binding, the first best is achieved and we have:

$$\varphi_w(w^*(\theta), \theta) = \lambda^* \quad \text{for any } \theta$$

$$\int_{\underline{\theta}}^{\bar{\theta}} w^*(\theta) dF(\theta) = \bar{w}$$

Suppose now that the first best is increasing so that constraint (15) is not binding.

The Hamiltonian is then:

$$(17) \quad H = \varphi(w(\theta), \theta) f(\theta) - \lambda w(\theta) f(\theta) + v_1(\theta) z(\theta) + v_2(\theta) \mu(\theta)$$

From the Pontryagin principle we get:

$$(18) \quad v_1 = (\lambda - \varphi'_w) f$$

$$(19) \quad v_2 = -v_1$$

with the transversality conditions:

$$(20) \quad v_1(\underline{\theta}) = v_1(\bar{\theta}) = v_2(\underline{\theta}) = v_2(\bar{\theta}) = 0$$

Maximization of (17) with respect to μ gives:

$$(21) \quad v_2 \leq 0 \quad \text{and} \quad \mu = 0 \quad \text{if} \quad v_2 < 0.$$

Whenever the optimal solution $\bar{w}(\cdot)$ is strictly convex on an interval it entails the same constant value φ_w . From (21), $\bar{w}'' > 0$ implies $\mu > 0$ which implies $v_2 = 0$ on this interval. (19) implies $v_1 = 0$ hence $\varphi'_w = \lambda$ on this interval.

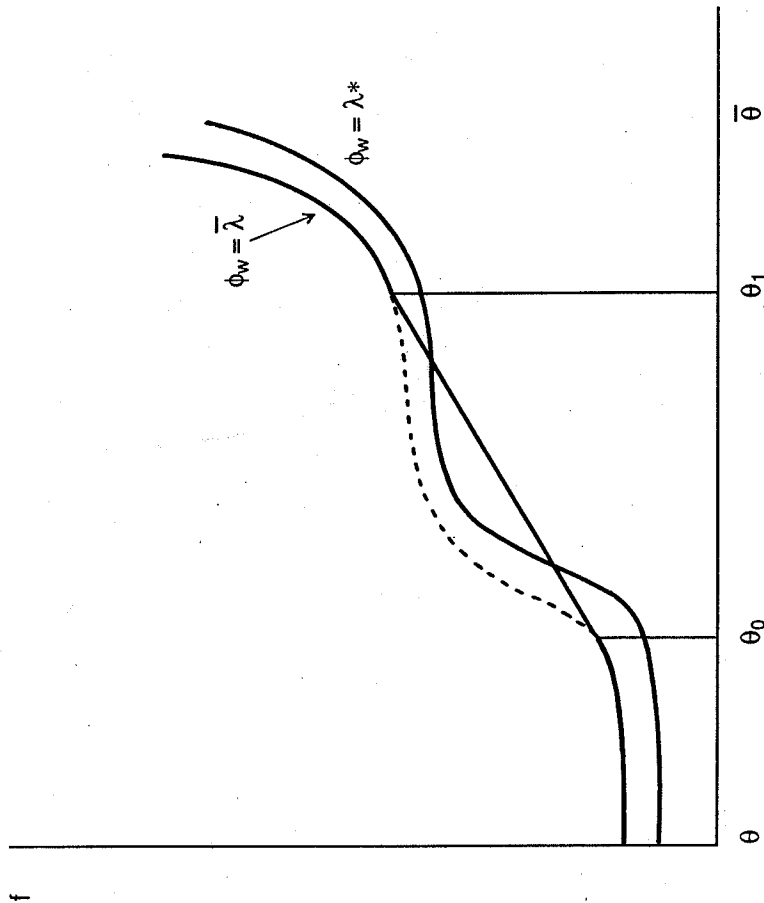
From (21) we see that when the solution is not strictly it is linear. Let θ_0, θ_1 , such an interval where it is not convex. Integrating (18) between θ_0 and θ_1 and using the continuity of the Pontryagin multipliers at θ_0 and θ_1 we have:

$$v_1(\theta_1) - v_1(\theta_0) = 0 = \int_{\theta_0}^{\theta_1} (\lambda - \varphi'_w) dF.$$

As the state variables are continuous we have

$$\bar{w}(\theta_0) = \bar{w}(\theta_1)$$

FIGURE 5



Observe that the linear piece in figure 5 can only be obtained from a simple contract \hat{x}, \hat{t} since \hat{x}, \hat{t} generates a linear utility level,

$$w(\theta) = \theta v(\hat{x}) - \hat{t}$$

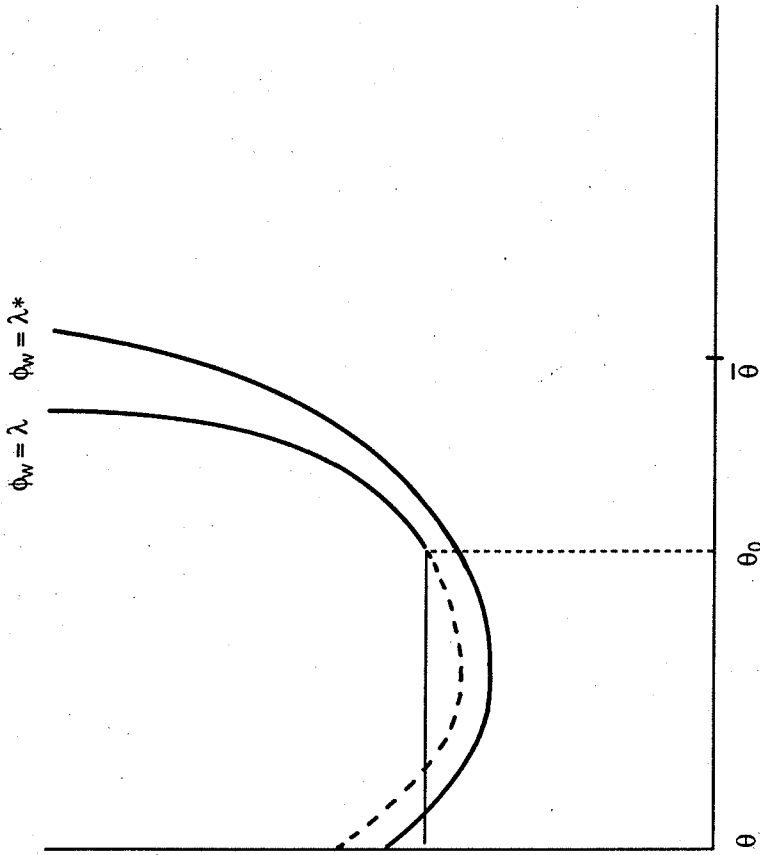
and if x were not constant $w(\theta)$ would not be linear.

Suppose now that the first best is convex but first decreasing and then increasing. Similar arguments as above show then that when it is increasing, the optimal solution is analogous to the first best with a different value of the multiplier and is flat elsewhere (see Figure 6). The interval $(\underline{\theta}, \theta_0)$ is determined by the condition:

$$\int_{\underline{\theta}}^{\theta_0} (\lambda - \varphi'_w) dF(\theta) = 0$$

When $v(0) = 0$, the constant piece has the interpretation of a cancellation of delivery ($x = 0$) that the agent selects with an associated transfer t determining the level of the payment. If $t < 0$ it can be interpreted as a cancellation fee that the agent must pay when he chooses not to transact. If $t > 0$ it can be interpreted as an hostage given by the principal to encourage

FIGURE 6



When both constraints (15), (16) may be binding the solution is more difficult to describe because the constraints on state variables may induce jumps at the optimal solution.

The question of implementability of first best allocations reduces then to the characterization of cases when the first best utility profile of the agent is increasing and convex.

An interesting result is obtained when the principal's utility function is:

$$(22) \quad f(t) = -x$$

We then have:

THEOREM 4: For the utility functions (14), (22) the first best allocation is implementable via delegation and P-led renegotiation iff the index of absolute risk aversion of v is decreasing.

Proof: In this case the first best is such that

Hence,

$$(24) \quad x^{*'} = -\frac{v'}{\theta v''}; \quad x^{*''} = \frac{2v'}{\theta^2 v''} - \frac{v^2 v'''}{\theta^2 v'^3}.$$

In the first best $f'(t^*(\theta)) = \lambda^*$; therefore t^* is constant. Differentiating twice the agent's utility function we obtain:

$$(25) \quad \frac{d^2 w^*(\theta)}{d\theta^2} = v'(x^*(\theta)) x^{*'}(\theta) + \theta v''(x^*(\theta)) (x^{*'}(\theta))^2 + \theta v'(x^*(\theta)) x^{*''}(\theta) + v'(x^*(\theta)) x^{*'}(\theta)$$

The differentiation of (23) shows that the first two terms of (25) cancel. Using (24) we obtain

$$\frac{d^2 w^*(\theta)}{d\theta^2} = -\lambda^* \frac{v'}{\theta v''} \frac{d}{dx} \left(-\frac{v'}{v''} \right) > 0.$$

Hence the result from theorem 3. \square

6 Examples

From section 2 we know that A-led renegotiation, being a special case of incentive compatible mechanism, is dominated by a contract of the type delegation and P-led renegotiation. However, authentic delegation may be necessary, *i.e.* A-led renegotiation may not be dominated by any simple contract with renegotiation. We provide below such an example.

Example 1

$$u(x, t, \theta) = -t + \theta v(x)$$

$$f(t, x) = f(t - x)$$

In the first best we have $f'(t^*(\theta) - x^*(\theta)) = \text{constant}$ and therefore $f(t^*(\theta) - x^*(\theta)) = \text{constant}$.

With a constant contract (\hat{x}, \hat{t}) followed by renegotiation, the principal solves:

$$\text{Max } f(t - x)$$

$$\theta v(x) - t \leq \theta v(\hat{x}) - \hat{t}$$

Proof: In this case the first best is such that

To implement the first best allocation with a simple contract with A-led renegotiation, the principal can choose \hat{x} , \hat{t} such that

$$f(\hat{t} - \hat{x}) = f(t^*(\theta) - x^*(\theta)) = \text{const.}$$

Then the agent solves:

$$\text{Max } -t + \theta v(x)$$

subject to

$$f(t - x) \geq f(t^*(\theta) - x^*(\theta))$$

and reaches the first best allocation.

It should be clear that the mechanism delegation and P-led renegotiation is more powerful than a contract with agent's control. We illustrate this point with the example used in theorem 4.

Example 2

$$u(x, t, \theta) = -t + \theta v(x)$$

$$f(t, x) = f(t) - x$$

Under the increasing absolute risk aversion of v the first best is reached by delegation and P-led renegotiation. As t^* is constant in the first best and x^* increasing the principal's utility level is decreasing in θ in the first best and we know that A-led renegotiation implements only constant utility levels for the principal. Indeed this latter mechanism is just a particular incentive compatible mechanism and here the first best is not incentive compatible and *requires renegotiation*. This is because whenever t^* is constant x^* must be constant to insure incentive compatibility.

On the contrary, in example 1 renegotiation is not really needed because the first best is incentive compatible. From efficiency,

$$f' = \lambda \quad \text{and} \quad f' = \lambda \theta v'(x)$$

Incentive compatibility requires:

$$-t' + \theta v'(x) x' = 0$$

or from efficiency

$$x' - t' = 0$$

But as in the first best the principal's utility is constant $x' = t'$.

Implementation of a given first best allocation must be distinguished from implementation of first best levels of utilities when first best allocations are not unique. The following example illustrates that implementation of a given first best allocation is much more demanding than implementation

Example 3

$$u(x, t, \theta) = -t + \theta v(x)$$

$$f(t, x) = t + f(x)$$

In a first best allocation $f'(x^*(\theta)) = \theta v'(x^*(\theta))$ but $t^*(\theta)$ is only constrained by

$$\int t^*(\theta) dF(\theta) = -\bar{w} + \int \theta v(x^*(\theta)) dF(\theta) = K$$

One particular first best allocation is then

$$t^*(\theta) = K \quad \forall \theta, \quad x^*(\theta) \text{ such that } f'(x) = \theta v'(x)$$

This allocation is implementable via delegation and P-led renegotiation iff

$$\frac{d^2}{d\theta^2} (-K + \theta v'(x^*(\theta))) \geq 0$$

or after a few manipulations

$$\frac{f''^2 - 2f''\theta v''}{(f'' - \theta v'')^2} + \frac{1}{(f'' - \theta v'')^3} \left[\theta v'^2 v''^2 \frac{d}{dx} \left(-\frac{v'}{v''} \right) + f'^2 \frac{d}{dx} \left(-\frac{f''}{f'} \right) \right] \geq 0$$

which obtains under our assumptions if $\frac{d}{dx} \left(-\frac{v'}{v''} \right) > 0$ (as in Theorem 4) and moreover if $\frac{d}{dx} \left(-\frac{f''}{f'} \right) > 0$.

However, first best levels of utilities are always implemented with A-led renegotiation. The principal chooses \hat{x} , \hat{t} such that

$$\hat{t} - f(\hat{x}) = \int [t^*(\theta) - f(x^*(\theta))] dF(\theta) = K$$

Then the agent selects the efficient $x^*(\theta)$ and chooses

$$t^*(\theta) = f(x^*(\theta)) - \int f(x^*(\theta)) dF(\theta) + K$$

which gives to him and to the principal the same expected utility level as in

Example 4⁴

$$u(x, t, \theta) = v(\theta x - t)$$
$$f(y, t) = t - f(x)$$

In this case the agent is risk averse in t as well as x , unlike cases 1-3 above. The first best requires

$$v_x = \theta v' = \lambda_x f'$$
$$v_t = -v' = \lambda$$

Achieving a fixed v' also produces a fixed u as a function of θ . Hence the first best is achieved by "no contract".

Notice that the normality conditions of theorem 2 fail in this case since

$$\frac{-u_t}{u_x} = -\frac{1}{\theta}$$

and hence

$$\frac{d}{dx} \left(\frac{-u_t}{u_x} \right) = \frac{d}{dt} \left(\frac{-u_t}{u_x} \right) = 0.$$

Moreover,

$$\frac{d}{dt} \left(\frac{-f_t}{f_x} \right) = \frac{d}{dt} \left(\frac{1}{f''(x)} \right) = 0$$

Conclusion

Observable but non-verifiable information is very common when long term bilateral relationships exist. Contracting in such cases is an attempt to capture the potential for risk sharing and efficient resource allocation. Mutually beneficial renegotiation represents a way to achieve *ex post* efficiency. But the common knowledge that contracts will be renegotiated may have adverse consequences for the efficient *ex ante* risk allocation.

In this paper we examine contracts that can specify or delegate authority over the collective decisions to one party and can give the other the right to improve upon this choice by making a take-it-or-leave-it offer in the renegotiation phase. Although more general contracts could be envisioned, these are the only ones that do not require the contract enforcement agency to be able to observe strategic interaction between the players.

We have shown how the choice of an *ex ante* contract works as an insurance device through its impact on the contingent individual rationality level of utility of the agent. Theorem 2 gave very general assumptions under which this insurance value of such an *ex ante* contract was always positive.

The second point of this paper has been to characterize, at least in one special case, the second best contract. We were then able to describe the optimal second best contracts and to show that they might incorporate cancellation fees or hostages. Finally conditions were found under which the first best could be reached.

Proof of Theorem 2

The proof proceeds constructively by defining a contract which involves A-led renegotiation and which dominates no contract. The contract will be denoted \hat{x}, \hat{t} . For each θ , the agent will give the principal the utility $f(\hat{x}, \hat{t})$ by choosing an *ex post* efficient $x(\theta), t(\theta)$. Thus, the significance of x, t is that it guarantees a level of utility for the principal. The specific pair \hat{x}, \hat{t} is not important. For each value of θ we will compare the utility level arrived at under the contract \hat{x}, \hat{t} with the utility levels achieved under no contract. In the former case, the principal gets $f(\hat{x}, \hat{t})$ and the agent gets the utility that corresponds to this level, given θ . In the latter case it is the agent's level that is invariant to θ , being \bar{w} , and the principal's realized utility will increase with θ .

We note that since the agent gets \bar{w} on average in either contract, he will realize above \bar{w} when θ is high and below \bar{w} when θ is low. The idea of the proof is to show that under the contract, when θ is low, the principal gains at the agent's expense and that this gain is larger than the loss the principal sustains when at, high θ , the agent's power in the renegotiation phase allows him to do better than \bar{w} . The method of proof is to examine the slope of the utility possibility set, for each θ , in the region between the points realized by these two contracts. We will show that, per unit change in the agent's utility, the principal's utility is more sensitive when θ is low than when θ is high. That is why the positive changes in the principal's utility outweigh the negative changes. We now proceed to this demonstration.

In the renegotiation phase, after the contract \hat{x}, \hat{t} has been signed, the agent will solve

$$(26) \quad \begin{aligned} &\max u(x, t, \theta) \\ &\text{subject to} \\ &f(x, t) \geq f(\hat{x}, \hat{t}) \end{aligned}$$

Let the value of this problem be denoted $\varphi(f(\hat{x}, \hat{t}), \theta)$.
Now consider the following system of equations:

$$(27) \quad f_x(\hat{x}, \hat{t}) + \lambda u_x(\hat{x}, \hat{t}, \hat{\theta}) = 0$$

$$(28) \quad f_t(\hat{x}, \hat{t}) + \lambda u_t(\hat{x}, \hat{t}, \hat{\theta}) = 0$$

$$(29) \quad u(\hat{x}, \hat{t}, \hat{\theta}) = \bar{w}$$

$$(30) \quad \int \varphi(f(\hat{x}, \hat{t}), \theta) dF(\theta) = \bar{w}$$

for the four unknowns $\hat{x}, \hat{t}, \hat{\theta}, \lambda$. Under some technical, but ordinary, conditions on the boundary behavior of f and u , the system has a unique solution.

The interpretation of these equations is as follows: Given \hat{x}, \hat{t} we know that the solution of (27-30) will generally result in $(x(\theta), t(\theta)) \neq (\hat{x}, \hat{t})$. However, there will be some value of θ , called $\hat{\theta}$, at which \hat{x}, \hat{t} is *ex post* efficient and therefore at which it remains in force after renegotiation. In addition we know that the level of utility $f(\hat{x}, \hat{t})$ that the principal proposes to guarantee himself in the contract will provide the agent with the *ex ante* utility \bar{w} , (30). Equation (29) says that this level of *ex ante* utility is realized *ex post* at precisely the same θ at which \hat{x}, \hat{t} is invariant to renegotiation.

This contract (\hat{x}, \hat{t}) is generally not efficient in the *ex ante* sense that it is not the best A-led contract. However it always dominates "no contract" as we now show.

Let us write the first-order conditions for the problem (26)

$$(31) \quad \mu f_x(x, t) + u_x(x, t, \theta) = 0$$

$$(32) \quad \mu f_t(x, t) + u_t(x, t, \theta) = 0$$

$$(33) \quad f(x, t) = f(\hat{x}, \hat{t})$$

where μ is the Lagrange multiplier of the constraint. We know that

$$\varphi_f(f(\hat{x}, \hat{t}), \theta) = -\mu(\theta)$$

and that $\mu(\theta) > 0$.

Let $\psi(u, \theta)$ be defined by

$$\psi(\varphi(f, \theta), \theta) = f, \quad \text{for all } f, \theta.$$

Then

$$\psi_u(u, \theta) = \frac{1}{\varphi_f(\psi(u, \theta), \theta)}$$

Let us compare the principal's utility under this contract to that obtained under no contract. The change in the principal's utility is

$$(34) \quad \int_{\bar{w}}^{\varphi(f(\hat{x}, \hat{t}), \hat{\theta})} \psi_u(u, \theta) du dF(\theta).$$

We can divide the integral over θ into two parts, above and below $\hat{\theta}$:

$$(35) \quad - \int_{\varphi(f(\hat{x}, \hat{t}), \hat{\theta})}^{\bar{w}} \psi_u(u, \theta) du dF(\theta) + \int_{\hat{\theta}}^{\bar{w}} \psi_u(u, \theta) du dF(\theta)$$

We show that in the range of the first double integral, where

$$u(x(\theta), t(\theta), \theta) < \bar{w} \quad \theta < \hat{\theta}$$

that

and, conversely, in the range of the second double integral where

$$u(x(\theta), t(\theta), \theta) > \bar{w} \\ \theta > \hat{\theta}$$

that

$$(37) \quad \Psi_u(u(x(\theta), t(\theta), \theta)) > -\hat{\lambda}$$

Note that

$$\mu(\hat{\theta}) = \frac{1}{\hat{\lambda}}$$

and that

$$\Psi_u(u, \theta) = \frac{1}{\varphi_r(\Psi(u, \theta), \theta)} = -\frac{1}{\mu(\theta)}$$

Therefore (36) and (37) will be proven if we can demonstrate that

$$\frac{d\mu(\theta)}{d\theta} < 0.$$

Differentiate the system (31-33) totally and apply Cramer's rule, obtaining

$$(38) \quad \Delta \frac{d\mu}{d\theta} = -u_{x\theta} (f_x(\mu f_{xt} + u_{xt}) - f_x(\mu f_{tt} + u_{tt}))$$

where

$$\Delta = \det \begin{bmatrix} \mu f_{xx} + u_{xx} & \mu f_{xt} + u_{xt} & f_x \\ \mu f_{xt} + u_{xt} & \mu f_{tt} + u_{tt} & f_t \\ f_x & f_t & 0 \end{bmatrix} < 0$$

$$+ u_{t\theta} (f_t(\mu f_{xx} + u_{xx}) - f_x(\mu f_{xt} + u_{xt}))$$

At the ex post efficient point, $\hat{\theta}$, we can use (31) and (32) to rewrite (38) as

$$\Delta \frac{d\mu}{d\theta} = -u_{x\theta} \left(\mu (f_x f_{xt} - f_x f_{tt}) - \frac{1}{\mu} (u_t u_{xt} - u_x u_{tt}) \right) \\ + u_{t\theta} \left(\mu (f_t f_{xx} - f_x f_{xt}) - \frac{1}{\mu} (u_t u_{xx} - u_x u_{xt}) \right)$$

and, under our normality condition (5-8), the right hand side of this equation is positive, hence $\frac{d\mu}{d\theta} < 0$, and hence (36) and (37) hold under the

Now we can bound (35) using (36),

$$(39) \quad - \int_{\hat{\theta}}^{\theta} \int_{\bar{w}}^{\bar{w}} \Psi_u(u, \theta) du dF(\theta) \\ \geq \int_{\hat{\theta}}^{\theta} \int_{\bar{w}}^{\bar{w}} \lambda du dF(\theta) \\ = -\lambda \int_{\hat{\theta}}^{\theta} [\varphi(f(\hat{x}, \hat{t}), \theta) - \bar{w}] dF(\theta)$$

In the range of the second double integral (37) implies

$$(40) \quad \int_{\hat{\theta}}^{\theta} \int_{\bar{w}}^{\bar{w}} \Psi_u(u, \theta) du dF(\theta) \\ \geq \int_{\hat{\theta}}^{\theta} \int_{\bar{w}}^{\bar{w}} (-\lambda) du dF(\theta) \\ = -\lambda \int_{\hat{\theta}}^{\theta} [\varphi(f(\hat{x}, \hat{t}), \theta) - \bar{w}] dF(\theta)$$

Combining the lower bounds (39) and (40) we have that (34) is bounded below by

$$-\lambda \int_{\hat{\theta}}^{\theta} \varphi(f(\hat{x}, \hat{t}, \theta) - \bar{w}) dF(\theta) \\ = 0.$$

by virtue of (30). \square

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