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COLLUSIVE PRICE LEADERSHIP*

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We study the pattern of pricing in which price changes are first announced by one firm and then matched by its rivals. In our model, this price leadership facilitates collusion under asymmetric information. In equilibrium the leader earns higher profits than the follower. Nonetheless, if information is sufficiently asymmetric, the less informed firm prefers to follow the better informed firm, so the leader can emerge endogenously. We show that the follower can benefit from price rigidity so that prices may be changed infrequently. We also show that overall welfare may be lower under collusive price leadership than under overt collusion.

I. INTRODUCTION

IN MANY industries pricing is characterized by price leadership: one of the firms announces a price change in advance of the date at which the new price will take effect and the new price and date are swiftly matched by the other firms in the industry. Price changes are often matched to the penny even when the products are differentiated, and it is common for a long time to elapse between price changes.

Examples of this pattern of pricing behavior abound. Perhaps the best known example occurred in the cigarette industry in the late 1920s and early 1930s. For instance, Reynolds announced an increase in its price from $6.00 to $6.40 per thousand on October 4, 1929 effective October 5. This move was followed the next day by both of its major competitors, Liggett and Meyers and American Tobacco. That price was in effect for almost two years before Reynolds led a further increase (to $6.85).[1] Similar pricing behavior has been documented in the steel, dynamite, anthracite and airline industries.[2]

Markham [1951] suggests that this pattern of pricing is "price leadership in lieu of overt collusion". Since meetings in "smoke-filled rooms" that result in price agreements violate the antitrust laws, firms are alleged to use these public announcements to achieve collusion. By contrast, Posner and Easterbrook [1981] claim that collusive price leadership is impossible. They argue that in any industry in which price leadership becomes established with prices in excess of marginal cost, any single firm can temporarily profit by

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[1] See Nicholls [1951] for a detailed discussion of pricing in the cigarette industry during this period.

[2] See Stigler [1947] for a discussion and further references.

cutting prices. Once its price cut has been detected and matched by its rivals the price cutting firm can simply lead the rivals back to the collusive price level.

In this paper, we show that collusive price leadership is certainly possible; Markham's view can be given coherent foundations. We also give conditions under which one would expect collusive price leadership to arise and study its theoretical properties. In particular we are concerned with its effect on price rigidity and welfare.

We examine a differentiated products duopoly in which the firms are asymmetrically informed. We investigate a price leadership scheme in which pricing decisions are delegated to the better informed firm (the "leader") which announces pricing decisions ahead of time. The leader then expects the follower to match these prices exactly.

This scheme has a number of positive attributes from the point of view of the duopoly: it is extremely easy to implement; defining adherence to the scheme is trivial in the sense that there is no ambiguity as to the desired response of the follower; no overt collusion (either through information transfer or price-fixing) is required and, while the scheme is generally not optimal, both firms enjoy responsiveness to demand conditions since prices embody the leader's superior information.

To capture the possible private benefits and social consequences of price leadership, we focus on a simple example with linear demand. We begin by providing the conditions under which such price leadership can be sustained as a collusive equilibrium in a repeated game by the (credible) threat that the industry will revert to uncooperative behavior if any firm "cheats" on the collusive understanding. The only issue is whether the follower will deviate. Since the leader picks the price he most desires for the industry, he has no incentive to deviate.

As is usual for games of this kind, the follower trades off the one-period gain from unilaterally deviating against the future costs of the breakdown in cooperation. However, the calculation of the incentive to deviate is complicated by the fact that the price announcement may reveal some of the leader's private information. As a result the follower's calculation involves a signal extraction problem.

Our main focus is on the characteristics of the equilibrium rather than the existence conditions, however. The main features of the equilibrium in our example are the following:

First, if the firms produce differentiated products and a common price is charged, they will typically have differing preferences about what that price should be. Since the firm that is the designated price leader typically earns higher profits under these circumstances, it might be expected that both firms would vie for the leadership position. We show, however, that when one of the firms is better informed than the other, the latter may prefer to follow than to lead. Thus the industry leader may emerge endogenously.

Second, the disparity in profits between the leader and follower can be reduced (and a more "equitable" distribution of profits achieved) if the price leader keeps its price constant for some time. Faced with the obligation to keep prices constant, the leader faces a tradeoff. On the one hand, it raises its own current profits if it exploits the follower by responding strongly to current relative demand. On the other hand, this strong response lowers future profits if relative demand conditions are expected to revert to normalcy. Therefore, rigid prices can reduce the response of the leader's price to current relative demand. Generally, however, the follower will not be in favor of completely rigid prices. Counterbalancing the profit-sharing benefit of inflexible prices is the advantage of letting the leader respond to common (and not merely relative) demand fluctuations. Somewhat (but not completely) rigid prices should therefore be expected to be a feature of a price leadership regime.

Third, social welfare (measured as the sum of consumer and producer surplus) is lower under price leadership than under overt collusion. While the duopolists are able to achieve a somewhat collusive outcome using price leadership, they clearly do not do as well as they would if they were completely unconstrained, i.e. if they could sign a binding contract and could enforce honest revelation of private information. Consumers, on the other hand, are generally better off under a price leadership regime than they are when faced with overt collusion. When demand is relatively strong for the leader's product, the leader sets a high price, and thus consumer surplus from each product is low compared with that under full collusion. The opposite is true when demand for the leader's product is relatively weak. However, the increase in consumer surplus when demand for the leader's product is relatively weak exceeds the reduction in consumer surplus when it is relatively strong. This is because consumer surplus is a convex function of price.

The paper treats the subjects in the following order: the necessary conditions for existence of a collusive price leadership equilibrium and the firms' unanimity in their choice of price leader (section II), price rigidity (section III), and welfare (section IV). Section V provides concluding remarks.

II. THE MODEL

We consider two firms. Firm 1 produces good 1 while firm 2 produces good 2. Each good is produced with constant marginal cost c. The demands for these goods are given by:

$$
\begin{aligned}
Q_1 &= x - bP_1 + d(P_2 - P_1) \\
Q_2 &= y - bP_2 + d(P_1 - P_2)
\end{aligned}
\tag{1}
$$

where Q_i and P_i are the quantity demanded and price of good i respectively. Note that, except for x and y, the demand for the two goods is symmetric. We consider this symmetric case so that we can focus on the asymmetries introduced by price leadership.

The constant b gives the response of the quantity demanded of each good to a decrease in the price of both goods while d gives the response to a relative increase in the other good's price. Thus d is a measure of the extent to which the two goods are substitutes; it is zero if the demand for both goods is independent while it is infinite if the goods are perfect substitutes.

The overall levels of demand for the two goods, represented by x and y, are assumed to fluctuate over time. Since, as we shall see, price leadership leads firms to respond differently to common and to idiosyncratic disturbances, it is useful to consider the new variables $a \equiv (x+y)/2$ and $e \equiv (x-y)/2$ which, substituting in (1), yields:

$$(2) \qquad \begin{aligned} Q_1 &= a+e-bP_1+d(P_2-P_1) \\ Q_1 &= a-e-bP_2+d(P_1-P_2) \end{aligned}$$

This change of variables gives a common disturbance a, which has the same effect on both demands, and an idiosyncratic disturbance e which raises the demand of 1 by the same amount as it reduces the demand for 2.[3] Assuming that the variances of x and y are the same, a and e are uncorrelated.[4] To maintain symmetry we also assume that the mean of e is zero. We denote the mean of a by a'.

We want to capture a situation in which the two firms have differential information about demand and in which one firm has somewhat better information. For simplicity, we make the extreme assumption that in each period firm 1 knows the realizations of both a and e while firm 2 knows only their distribution as well as the history of prices and quantities. In the simple case, which we consider in this section, where a and e are independently distributed over time, the history of prices and quantities is not informative about the current values of a and e. Therefore all of firm 2's information is summarized by the unconditional means of a and e (a' and 0 respectively).

To ensure that firm 2 doesn't receive additional information from firm 1 we also assume that there is no credible way for firm 1 to communicate its information.[5] While these informational assumptions are extreme, our main results carry over to more realistic environments. In particular, in Rotemberg

[3] We assume that demand varies while marginal cost is constant. However, similar results would follow if we let marginal costs vary. Then, e would represent increases in the marginal cost of firm 2 which are matched by equal reductions in the marginal cost of 1.

[4] To see this, denote the means of a and e by a' and e' respectively. Then:

$$\begin{aligned} \operatorname{cov}(a,e) &= E(a-a')(e-e') = E[x+y-(x'+y')][(x-y-(x'-y')]/4 \\ &= E[(x-x')+(y-y')][(x-x')-(y-y')]/4 = E[(x-x')^2-(y-y')^2]/4 \\ &= [\operatorname{var}(x)-\operatorname{var}(y)]/4. \end{aligned}$$

[5] Note that firm 1 has an incentive to lie about its information. If it could convince firm 2 that its demand is large, it could induce it to charge a high price thus increasing demand for its own product.

and Saloner [1985], we treat the case where both have access to some information about the realizations of a and e.

We suppose that the duopolists interact repeatedly. It is well-known that in repeated game settings firms may be able to achieve supracompetitive outcomes which are sustained in equilibrium by the credible threat that a price war will break out if any firm deviates from collusive behavior.[6]

With asymmetric information the firms face the issue of what price to collude on. In what follows we examine the "solution" in which the informed firm sets the price for all the industry members. Specifically, we suppose that, provided firm 2 has not cheated in the past, each period opens with firm 1 announcing its price for that period.[7] After that, firm 2 is expected to announce the same price for the period.

If firm 2 ever deviates and announces a different price then a price war ensues. For simplicity we suppose that the price war is of infinite duration and takes the form of the firms simultaneously announcing prices each period.[8] The prices they announce in all periods after a deviation by firm 2 are those that constitute equilibrium prices in a one shot game.

In this proposed equilibrium, since firm 1 is sure to be followed when it chooses a price equal to P, its per period payoff is:

$$R_1 = (P-c)(a+e-bP) \tag{3}$$

Since firm 1 picks its price to maximize (3),

$$P = c/2+(a+e)/2b$$

and firm 1's profits in equilibrium are given by:

$$R_1 = [a+e-bc]^2/4b \tag{4}$$

If firm 2 follows, its equilibrium profits are:

$$R_2 = [a+e-bc]^2/4b-[a+e-bc]e/b \tag{5}$$

Adding together (4) and (5), aggregate industry profits are:

$$\begin{aligned} R &= [a+e-bc]^2/2b-2[a+e-bc]e/2b \\ &= [a+e-bc][a-e-bc]/2b \\ &= [a-bc]^2/2b-e^2/2b \end{aligned} \tag{6}$$

Note from (4) and (5) that if e is always equal to zero the profits of the two firms are the same. In this case firm 1 picks the price which maximizes industry profits. But, whenever e is nonzero firm 1 picks prices which raise firm 1's profits at the expense of overall industry profits. As a result, since the mean of e is zero and e is uncorrelated with a, the average values of (5) and (6)

[6] For example, see Friedman [1971].

[7] Firm 1 cannot change its price for the period once it has made this announcement.

[8] This simplifies the analysis without affecting the conclusions.

decline as the variance of e rises, i.e. firm 2's profits and industry profits are decreasing in the variance of e.

We now consider firm 2's incentive to deviate. Conceptually, firm 2 wishes to deviate for two reasons. First, even when e is a constant equal to zero, firm 2 can increase its profits by undercutting firm 1. In addition, firm 2 generally wants to charge a price different from that chosen by firm 1 because it knows that firm 1's price increases with e while firm 2 would like to have a price that falls when e increases. This second motivation is somewhat complex because firm 2 does not know the value of e. It can nonetheless make an inference about e from the price chosen by firm 1. Then, the extent to which firm 2 wishes to charge a price different from the one charged by 1, depends on firm 1's actual announcement.

To compute whether firm 2 wishes to deviate we have to compute four different quantities. First, we calculate Z_2 the profits firm 2 can expect to earn in the future if it does not deviate in the current period. When a and e are independently distributed over time this is simply given by the unconditional expectation (5). Second, we calculate π_2, firm 2's expected profits in future periods if it does deviate in the current period. The per period value of the punishment from deviating is then $R_2-\pi_2$. Third, we calculate π_{D0}, the current period profit to firm 2 if it deviates. Finally, we calculate π_{C0} the current period profit to firm 2 if it does not deviate. This is simply the expectation of (5) conditional on firm 1's announcement. For firm 2 to be willing to match firm 1's price in equilibrium, the following condition must hold:

$$\pi_{C0}+\delta R_2/(1-\delta) \geqslant \pi_{D0}+\delta\pi_2/(1-\delta) \tag{7}$$

where δ is the discount factor. We compute π_{C0}, Z_2, π_{D0}, and π_2 in Appendix A where we also derive an expression for (7) in terms of the underlying parameters of the model.

It is now easy to see why collusive price leadership is possible in our model even though Posner and Easterbrook [1981] assert its logical impossibility. They assume that a deviating firm can always lead the industry back to the collusive price level after its deviation has been discovered, and matched. In our subgame perfect equilibrium this is impossible, the deviating firm is punished for its action by a long period of low prices. A price leadership equilibrium can presumably only emerge with the implicit consent of firm 2. But, comparison of the expectation of (4) with the expectation of (5) immediately reveals that firm 1 ends up with higher average profits than firm 2. So, the issue remains, does firm 2 prefer some other equally simple implicitly collusive equilibrium?

We thus compare firm 2's profits when firm 1 is the leader to those it obtains when it is, itself, the leader.[9] If firm 2's profits are higher when firm 1

[9] In this particular model, firm 2 is uninformed, so this is equivalent to having the duopoly agree on state independent prices.

is the leader, then, since their preferences coincide, it is natural to expect firm 1 to emerge endogenously as the leader.

If firm 2 is the price leader, it sets a price equal to $(a'+bc)/2b$ and its expected profits are given by:

$$L_2 = [a'-bc]^2/4b$$

Thus the difference between Z_2, firm 2's profits when firm 1 leads, and L_2, firm 2's profits when it leads, is simply

$$[E(a-a')^2 - 3Ee^2]/4b$$

Therefore, firm 2 prefers to be a follower if the variance of a exceeds three times the variance of e. A high variance of a makes firm 2 want to be a follower since movements in a are incorporated into firm 1's prices. On the other hand variations in e are also incorporated in firm 1's prices, to firm 2's detriment. So a high variance of e makes firm 2 prefer to be a leader.

Qualitatively similar conclusions emerge if both firms are somewhat (although asymmetrically) informed. In that case one can derive a similar condition for when the better informed firm is the unanimous choice as price leader.[10]

We conclude that price leadership has some natural advantages as a way of organizing a duopoly with asymmetric information. Not only does the leader have no incentive to deviate from its equilibrium strategy, but the detection of cheating by the follower is extremely simple. Moreover, in some circumstances the firms are unanimous in their choice of a leader.

Nonetheless it is important to note that the price leadership scheme does not replicate what the firms would be able to achieve under full information. Maximization of industry profits would dictate charging prices P_1 and P_2 which maximize $[Q_1(P_1-c)+Q_2(P_2-c)]$. Using (1) these prices are equal to $[a/b+c+e/(b+2d)]/2$ and $[a/b+c-e/(b+2d)]/2$ respectively. These prices are different as is to be expected. Total industry profits, U, are:

$$U = [a-bc]^2/2b + e^2/(2b+4d) \tag{8}$$

which is obviously larger than the joint profits under price leadership given by (6) (unless e is identically equal to zero).

While maximization of industry profits should probably be regarded as too ambitious a goal for a duopoly which is unable to make side payments, price leadership also falls short of being Pareto optimal from the point of view of the two firms.

This lack of Pareto optimality can be seen with the aid of Figure 1 which shows isoprofit lines for both firms in the space of P_1 and P_2. These lines are drawn for $e = e'' > 0$. Therefore the tangency of an isoprofit line for firm 1 and the 45° line occurs at prices higher than the tangency of an isoprofit line

[10] The details of this analysis are contained in our working paper, Rotemberg and Saloner [1985].

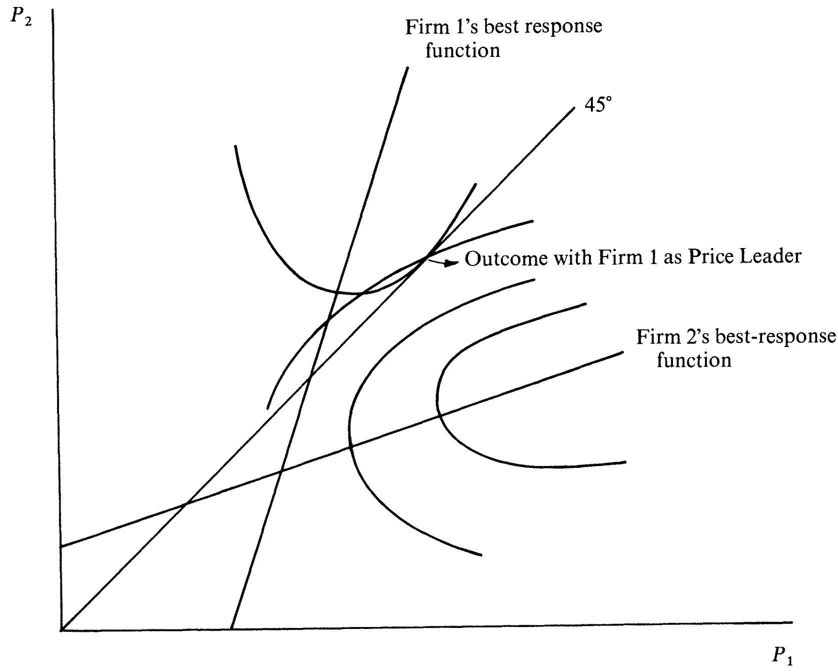

Figure 1
The Suboptimality of Price Leadership Under Full Information.

of firm 2 and the 45° line. In our model of price leadership firm 1 picks the point at which one of its isoprofit lines is tangent to the 45° line. It is immediately apparent from the figure that both firms can be made better off if they lower their prices, with the reduction of firm 2's price exceeding firm 1's.

In conclusion, price leadership achieves the optimal response to common changes in demand with great ease. Its cost (from the firms' point of view) is that prices do not respond optimally to changes in relative demand.

III. PRICE STICKINESS AND PRICE LEADERSHIP

The problem, from firm 2's perspective, in allowing firm 1 to be the leader, is that firm 1 takes advantage of this and picks a price that rises proportionately to e, the difference in the two demand curves. This exploitation comes about not only when demands differ, but also when costs differ, as when firm 1 faces a strike by its workers. One way of mitigating this effect, particularly when the firms possess information of similar quality, is to let the firms alternate the leadership role.[11] An alternative way, and one that is more applicable when the firms' quality of information differs substantially and when temporary

[11] Alternation of this kind has been observed in a variety of industries, including steel and cigarettes.

fluctuations in e are important, is to make prices relatively rigid. In other words, the leader is threatened with reversion to non-cooperation if it changes its price too often. We study this role for rigid prices here.

If the leader must keep its price fixed for some time, it will make its price a function of current and expected future es. The longer the period of price rigidity, the more important are the expected future es when firm 1 sets its price. Accordingly, if the expectation of future es is relatively insensitive to current demand conditions, the presence of rigid prices dampens the effect of current e on price.

We illustrate this advantage of price rigidity with a simple example. In particular, we assume that e_t, i.e. the value of e at time t, is given by:

$$e_t = \beta e_{t-1} + \varepsilon_t$$

where β is a number between zero and one while ε_t is an i.i.d. random variable with zero mean. The value of a at time t has two components; the first of which is a', a constant, while the second, a_t, moves over time according to the law of motion:

$$a_t = \phi a_{t-1} + \alpha_t$$

where ϕ is a number between zero and 1 while α is an i.i.d. random variable with zero mean. Thus the common component of demand, $a' + a_t$, tends to return to its normal value a' as well. To accommodate the existence of changes in the price level we write the demand curve at t as:

$$Q_{1t} = a' + a_t + e_t - bP_{1t}/S_t + d(P_{2t} - P_{1t})/S_t$$

$$Q_{2t} = a' + a_t - e_t - bP_{2t}/S_t + d(P_{1t} - P_{2t})/S_t$$

where S_t, the price level at t, is given by:

$$S_t = \mu^t$$

The difference between μ and one is the general rate of inflation. If the price leader must set a price that will be in force for n periods starting at time zero, it will pick a price that maximizes:

$$E_{10} \sum_{t=0}^{n} \delta^t \left(\frac{P}{S_t} - c\right)\left(a_t + a' + e_t - b\frac{P}{S_t}\right)$$

$$= \sum_{t=0}^{n} \delta^t \left(\frac{P_t}{\mu^t} - c\right)\left(\phi_t a_0 + a' + \beta^t e_0 - b\frac{P}{\mu^t}\right)$$

where E_{10} is the expectation conditional on information available at time 0 to firm one and δ is the real discount rate. This price, $P(n)$ is given by:

$$P(n) = \frac{\dfrac{a_0}{2b}\dfrac{1-(\delta\phi/\mu)^{n+1}}{1-\delta\phi/\mu} + \dfrac{e_0}{2b}\dfrac{1-(\delta\beta/\mu)^{n+1}}{1-\delta\beta/\mu} + \left(\dfrac{a'}{2b} + \dfrac{c}{2}\right)\dfrac{1-(\delta/\mu)^{n+1}}{1-\delta/\mu}}{\dfrac{(1-\delta/\mu^2)^{n+1}}{1-\delta/\mu^2}}$$

Note that $P(n)$ is increasing in n if there is inflation (μ is greater than one). It is also increasing in μ and e.

The expectation of the present discounted value of profits of the follower ($W_2(n)$) is given by the expectation of the present value of profits for the n periods during which the leader keeps its price fixed ($\pi(n)$) divided by $(1-\delta^{n+1})$. $\pi(n)$ is given by:

$$\pi(n) = E_0 \sum_{t=0}^{n} \delta^t \left(\frac{P(n)}{\mu^t} - c\right)\left(a' + \phi^t a_0 - \beta^t e_0 - b\frac{P(n)}{\mu^t}\right)$$

where E_0 takes unconditional expectations. The unconditional expectation of both a_0 and e_0 is zero while their unconditional variance is $\operatorname{var}(\alpha)/(1-\phi)$ and $\operatorname{var}(\varepsilon)/(1-\beta)$ respectively. This focus on unconditional expectations is warranted if the follower is completely uncertain about the state of demand at the moment it accepts the role of follower.

We can now write $W_2(n)$ as:

$$\begin{aligned} W_2(n) = {} & \frac{[a'+cb]^2[1-\delta/\mu^2][1-(\delta/\mu)^{n+1}]^2}{4b[1-\delta/\mu]^2[1-(\delta/\mu^2)^{n+1}[1-\delta^{n+1}]} - \frac{a'c}{1-\delta} \\ & + \frac{\operatorname{var}(\alpha)[1-\delta/\mu^2][1-(\delta\phi/\mu)^{n+1}]^2}{4b(1-\phi)[1-\delta\phi/\mu]^2[1-(\delta/\mu^2)^{n+1}][1-\delta^{n+1}]} \qquad (9) \\ & - \frac{3\operatorname{var}(\varepsilon)[1-\delta/\mu^2][1-(\delta\beta/\mu)^{n+1}]^2}{4b(1-\beta)[1-\delta\beta/\mu]^2[1-\beta/\mu^2]^{n+1}[1-\delta^{n+1}]} \end{aligned}$$

To evaluate the benefits of price rigidity we consider two special cases. In the first, μ is one (so that there is no inflation) while in the second μ differs from one but the variance of α is zero so that demand for the sum of the two products is deterministic.[12] In both cases there is an incentive to prolong the duration of prices, because this prolongation reduces the deleterious effect of $\operatorname{var}(\varepsilon)$ on W_2. In both cases there is also a cost to long price durations. In the first case this cost is the lack of adjustment to changes in the price level while in the second it is the insufficient response to changes in a. We analyze these special cases as follows. First, we give the conditions under which the follower would prefer some price rigidity, i.e. under which $W_2(2) > W_2(1)$. Then we study the numerical properties of $W_2(n)$ for certain parameters.

Consider first the case of a constant price level in which μ equals one. The first term of (9) is then independent of n. Assume also that $\operatorname{var}(\alpha)/[(1-\phi)(1-\delta\phi)^2]$, which we denote σ_a, equals $3\operatorname{var}(\varepsilon)/[(1-\beta)(1-\delta\beta)^2]$, which we denote σ_e. Recall that this is the condition under which firm 2 is just indifferent between being a follower and a leader. Then, the follower prefers prices to be constant for two periods if:

[12] Of course, in a model in which the variance of α is literally zero, our rationale for price leadership disappears. However, this example is only intended to illustrate the effects of inflation on price rigidity.

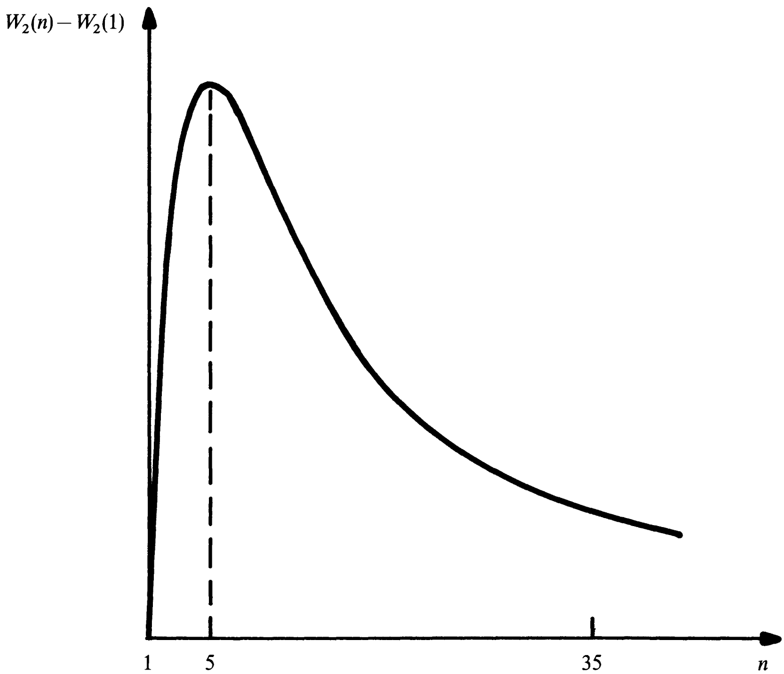

Figure 2
Profitability of Price Stickiness for the Follower—The Case of Stochastic Aggregate Demand.

$$\frac{[1-(\delta\phi)^2]^2}{1-\delta^2} - \frac{[1-(\delta\beta)^2]^2}{1-\delta^2} > \frac{[1-\delta\phi]^2}{1-\delta} - \frac{[1-\delta\beta]^2}{1-\delta} \tag{10}$$

As shown in Appendix B this inequality is satisfied as long as β is less than ϕ. Thus as long as the decay towards zero of the difference in demands is more rapid than the decay of the absolute level of demand towards its normal value, the follower prefers the leader to maintain some price rigidity. The intuition for this is that a rapid decay of e towards zero means that the leader will be relatively inattentive to e when setting a price for a relatively long horizon. On the other hand, if a decays slowly, the leader will still make its price fairly responsive to the current value of a.

We now show that the follower may prefer a finite period of price rigidity to an infinite one. This is plausible since, if e decays rapidly, the benefit from continued price rigidity, namely the loss in responsiveness to e becomes unimportant as the horizon becomes longer. We provide a numerical example in which the follower does indeed prefer a finite period of price rigidity.[13]

[13] This preference for finite periods of price rigidity is a feature of every numerical example we have studied.

Figure 2 shows the value of $W_2(n)$ when σ_a equals σ_e while k is 0.98, β is 0.6 and ϕ is 0.9. The follower's welfare is maximized when n is equal to five. If, instead, σ_a is made to equal only $0.8\sigma_e$ then the maximum occurs at $n = 7$. Clearly, an increase in the variance of the difference in demands warrants a longer period of price rigidity to reduce further the effect of e on price.

Now consider the special case in which the var(α) is zero. Assume further that, σ_e is equal to $(a' + cb)^2/4b$ which we denote σ_μ.[14] Suppose that β is one. Then, the first and third term of (9) are equal so the whole expression is independent of n. The follower here loses from the lack of responsiveness to the price level exactly what he gains from the lack of responsiveness to e. He is thus indifferent to the length of the interval over which prices are fixed.

If, instead, β is less than one, it can be shown that the follower always prefers some price rigidity. Indeed, it can be shown then that $W_2(2) > W_2(1)$. This requires that:

$$\text{(11)} \qquad \frac{[1-(\delta/\mu)^2]^2 - [1-\delta\beta/\mu]^2}{(1-\delta^2/\mu^4)(1-\delta^2)} > \frac{(1-\delta/\mu)^2 - (1-\delta\beta/\mu)^2}{(1-\delta/\mu^2)(1-\delta)}$$

which is proved in Appendix B for the case in which μ exceeds one. An analogous argument proves that (11) holds for the deflation case where μ is less than one. Thus, if the variance of ε is sufficiently big while e decays even slightly towards its mean, firm 2 prefers some rigidity to complete flexibility. Once again, a decay of e over time induces the leader to make its price unresponsive to e if it is to keep a relatively rigid price. The follower benefits from this. We again consider some numerical examples to show that the follower may prefer a finite period of price rigidity.

Figure 3 shows $W_2(n)$ for δ of 0.98, β of 0.7, μ of 1.02 and σ_e equal to one fourth of σ_μ. The maximum is given by $n = 18$. Reductions in inflation that make μ equal to 1.01 raise the optimal n, from the follower's perspective, to 25. Increases in σ_e also tend to raise this optimal n.

IV. WELFARE CONSEQUENCES OF PRICE LEADERSHIP

Given that the price leadership is an imperfect collusive device, one might think that it is not as bad from the point of view of overall welfare as overt collusion. In this section we show that this is not true in our model. We saw in section II that price leadership gives lower profits than overt collusion. In this section we show that, by comparison with overt collusion, price leadership is better for consumers in that model. Nonetheless overall welfare (as measured by summing producer and consumer surplus) is lower under price leadership.[15]

[14] This is probably unrealistic since it requires a very large variance of ε.

[15] While we demonstrate these results for the model of section II, the results hold also for the model of section III since the proof requires only that the unconditional mean of e be zero.

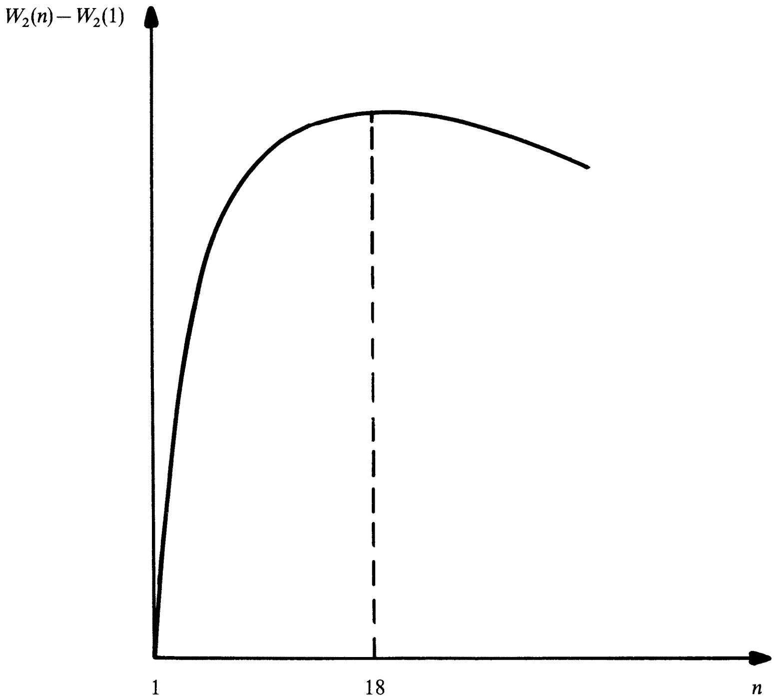

Figure 3
Profitability of Price Stickiness for the Follower—The Case of Inflation.

In principle the analysis of consumer welfare should be carried out using the preferences of consumers whose aggregate demands are given by (1). In practice this is very difficult, in part because different individuals will be affected differently, thus mandating interpersonal comparisons of utility, and in part because modelling the individual consumers whose demands aggregate to (1) is non-trivial. Therefore we compare instead the expected consumer surplus under the two regimes.

To simplify further we neglect variations in a and set c equal to zero. For this case the sum of producer and consumer surplus is the total area under the appropriate inverse demand curves. From (2) the inverse demand curves are given by:

$$P_1 = [a/b + e/(b+2d)] - Q_1(b+d)/[2(b+2d)] - Q_2 d/[2b(b+2d)]$$

$$P_2 = [a/b - e/(b+2d)] - Q_2(b+d)/[2(b+2d)] - Q_1 d/[2b(b+2d)]$$

To obtain the change in welfare from one equilibrium to another, i.e. from one pair of quantities to another, one integrates:

$$\int P_1(Q_1, Q_2)dQ_1 + P_2(Q_1, Q_2)dQ_2$$

on a line from the first pair of quantities to the other. For our particular demand curves (and in general when compensated demand curves are used) the actual path of integration is irrelevant.[16]

When firms collude overtly, Q_1 and Q_2 maximize $Q_1P_1+Q_2P_2$. Using (2) it is apparent that the outputs that maximize aggregate profits are given by $(a+e)/2$ and $(a-e)/2$ for firms 1 and 2 respectively. Under price leadership the corresponding quantities are instead $(a+e)/2$ and $(a-3e)/2$. Thus the output of the leader is actually equal to its output under overt collusion and only the output of the follower is different. The effect of price leadership is to make the follower's output very responsive to its demand. Thus it is very high when demand is high (and e is very negative) and very low when demand is low (e high).

To compute the change in welfare resulting from the replacement of overt collusion with price leadership for a given e, it is thus enough to integrate under the inverse demand curve for good 2 from $(a-e)/2$ to $(a-3e)/2$ holding Q_1 at $(a+e)/2$. This gives:

$$-ae/2b-de^2/[2b(b+2d)] \tag{12}$$

The first term of (12) is linear in e; it leads to losses from price leadership when e is negative and gains when it is positive. Given that the mean of e is zero, however, this term has no effect on the average difference between welfare in the two regimes.

The second term is quadratic. As e becomes larger output of good 2 continues falling under leadership and the marginal units lost become more and more valuable. So price leadership becomes more than proportionately deleterious. This second term is negative for all nonzero realizations of e. Thus, (12) is negative on average and price leadership leads to lower average welfare.

The social losses from price leadership can be interpreted in another way. On average, follower output equals $a/2$ both under leadership and under collusion. The main effect of leadership is to amplify the fluctuations in follower output. Since welfare tends to be a convex function of output, it declines on average as a result of these fluctuations.

While overall welfare is lower, we now show that consumers are better off. To do this it suffices to show that the decrease in profits from moving to price leadership exceeds the decrease in overall welfare. The difference in profits is given by the difference between (8) and (6) and equals:

$$e^2[1/b+1/(b+2d)]/2$$

which is larger than the loss in overall welfare (12) once one ignores the term linear in e.

[16] This occurs because our inverse demand functions are the partial derivatives of a function of Q_1 and Q_2. In our case that function is quadratic. See Diamond and McFadden [1974] for a general discussion.

From the point of view of consumers, price leadership has the disadvantage that, when e is high, both prices are high so there is little surplus in either market. On the other hand when e is negative, both prices are low so surplus is high particularly in the market for the preferred good (good 2). This latter effect dominates because the reduction in the price of good 2 when good 2 is the preferred good (which is given by $e(b+d)/[b(b+2d)]$) is larger than the increase in the price of good 1 when good 1 is the preferred good (which is given by $ed/[b(b+2d)]$).

V. CONCLUDING REMARKS

This paper has explored the possibility that price leadership is a collusive device in industries in which firms have private information. In such a setting price announcements go part way towards providing the kind of information revelation the firms could achieve if they could meet to share information and fix prices. The key features of the model are these: firms may be unanimous in their choice of price leader even though the price leader is in an advantageous position; price rigidity may serve as a device for decreasing the dispersion in the profits of the firms; and, from a welfare point of view, price leadership may be worse than overt collusion.

Two other forms of price leadership are discussed in the Industrial Organization literature. The first, "barometric" price leadership, refers to a situation in which the price leader merely announces the price that would, in any event, prevail under competition. In contrast to the collusive price leader, the barometric price leader has no power to (substantially) affect the price that is charged generally in the industry. Indeed the actual price being charged may soon diverge from that announced by the barometric firm, which in turn is unable to exert any disciplining influence to prevent this from occurring. When price leadership involves matching prices to the penny in markets where products are differentiated, the barometric model therefore does not seem very persuasive.

The other form of price leadership, and the one which has been the focus of most formal modelling, is the one that results from the existence of a dominant firm. Models of this type (see Gaskins [1971] and Judd and Petersen [1986]) assume that the dominant firm sets the price of a homogeneous product. This price is then taken as given by a competitive fringe of firms. Unfortunately, this model cannot explain the behavior of oligopolies in which there are several large firms. Such large firms cannot be assumed to take as given the price set by any one firm. Rather, they should be expected to act strategically.[17]

[17] The identical criticism can be applied to the model of d'Aspremont *et al.* [1983]. There, a group of equal-sized firms collude to set the price; the remaining firms, which are assumed to be of the same size, treat this price parametrically. Since this explicitly assumes that the fringe firms are large, the above criticism is especially relevant.

Understanding which of these three forms of price leadership is empirically most relevant is clearly important. Before closing it is thus worth recapitulating the features of price leadership predicted by our model. First, in our model price leadership raises profits. Second, the profits of the leader tend to be higher than those of its followers. Finally, the leader tends to be a firm with superior information. One avenue for empirical research is thus to uncover whether these three features are common to episodes of price leadership.

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APPENDIX A

NECESSARY CONDITION FOR PRICE LEADERSHIP TO BE AN EQUILIBRIUM

Here we compute condition (7) in terms of the underlying parameters of the model by first computing Z_2, π_2, π_{D0} and π_{C0}.

(*i*). Z_2

This is simply the unconditional expectation of (4) which equals:

$$Z_2 = E[a-bc]^2/4b - 3Ee^2/4b$$

(*ii*). π_2

Since firm 2 has no state dependent information and prices are announced simultaneously, it always announces the same price P_2. Thus firm 1 maximizes:

$$\pi_1 = (P_1 - c)(a + e - bP_1 + d(P_2 - P_1))$$

which leads to a price P_1 equal to $(a+e+dP_2+bc+dc)/2(b+d)$. Firm 2, on the other hand, maximizes:

$$\pi_2 = E(P_2 - c)(a - e - bP_2 + d(P_1 - P_2))$$

where E is the expectations operator and π_2 is the expected profit of firm 2. Thus firm 2 charges:

$$P_2 = (a' + bc + dc + dEP_1)/2(b+d) = (a' + (b+d)c)/(2b+d)$$

This implies that firm 1 charges:

$$P_1 = (a' + (b+d)c)/(2b+d) + (a - a' + e)/2(b+d)$$

The equilibrium value of π_2 is:

$$\text{(A1)} \qquad \begin{aligned} \pi_2 &= [(a' + (b+d)c)/(2b+d) - c][a' - b(a' + (b+d)c)/(2b+d)] \\ &= (b+d)[(a' - bc)/(2b+d)]^2 \end{aligned}$$

Note that the first term in brackets in (A1) represents the difference between price and marginal cost while the second represents the average output of firm 2. For both of these magnitudes to be positive a' must exceed bc, which, in turn, must be positive if coordinated price increases are to reduce industry sales.

If a and e are i.i.d., the cost to firm 2 of the punishment is that it loses $R_2-\pi_2$ in every period starting with the next one. The discounted present value of these losses equals:

$$D' = \{[E(a-a')^2-3Ee^2]/4b+(a'-bc)^2d^2/[4(2b+d)^2b]\}\delta/(1-\delta).$$

As shown in the text, the first term in brackets actually represents the advantage of letting firm 1 be the leader instead of firm 2. The second term gives the excess of collusive over competitive profits when e is zero and a equals a'. D' is the difference in profits between being a follower in our price leadership model and refusing to cooperate. It must be positive for price leadership to be in both firms' interests.

(iii). π_{D0}

After observing P, firm 2 becomes somewhat informed about $(a-a')$ and e since it has an indirect observation of $(a-a'+e)$. Indeed, it knows that $[2bP-bc-a'] \equiv x$ is equal to $(a-a'+e)$. Calculating its profits in the current period (whether from deviating or matching) therefore involves solving a signal extraction problem. Let s_a be the variance of a while s_e is the variance of e. Then, firm 2's expectation of $(a-a')$ is equal to $xs_a/(s_a+s_e)$ while its expectation of e is $xs_e/(s_a+s_e)$. If it were to charge P_2 after firm 1 irrevocably announced the price given by (3) its expected profits would be:

$$\text{(A2)} \qquad (P_2-c)\{a'+[2bP-bc-a'](s_a-s_e)/(s_a+s_e)-bP_2+d(P-P_2)\}.$$

If it deviates, firm 2 maximizes (7) which gives a price P'_2:

$$\begin{aligned} P'_2 &= \{a'+[2bP-bc-a'](s_a-s_e)/(s_a+s_e)+dP+(b+d)c\}/2(b+d) \\ &= \{2s_e a'+[bs_a+(b+d)(s_a s_e)]P+[ds_a+(2b+d)s_e]c\}/2(b+d)(s_a+s_e) \end{aligned}$$

and expected profits of $\pi_{D0} \equiv (b+d)(P'_2-c)^2$. Since firm 2 tends to profit by undercutting firm 1, P'_2 tends to be below P. As long as the variance of e is low enough, this is true for all P.

(iv). π_{C0}

On the other hand, by not deviating, firm 2 earns the expectation of (A2) evaluated at P. This is the expectation of $(P-c)(a-e-bP)$ conditional on P, which equals:

$$\text{(A3)} \qquad (P-c)[a's_e+b(s_a-3s_e)P-bc(s_a-s_e)]/(s_a+s_e)$$

Substituting for $\pi_C{}^0$, R_2, π_{D0}, *and* π_2 in (6) gives the key necessary condition for existence of a collusive equilibrium:

$$(b+d)(P'_2-c)^2-\frac{(P-c)[a's_e+b(s_a-3s_e)P-bc(s_a-s_e)]}{s_a+s_e}-D' < 0$$

This condition assumes that firm 1 punishes firm 2 both for downwards and upwards deviations in its price. Punishment for upwards deviations seems unreasonable. As mentioned above however, under plausible conditions firm 2 always undercuts firm 1 when it deviates.

APPENDIX B

PROOF OF INEQUALITIES 10 AND 11

Since, δ, β and ϕ are less than one inequality (10) can be written as follows:

$$\frac{(\delta\beta)^2-(\delta\phi)^2}{1-\delta^2} > \frac{\delta\beta-\delta\phi}{1-\delta}X$$

where

$$X = \frac{(2-\delta\phi-\delta\beta)(1+\delta)}{2-(\delta\phi)^2-(\delta\beta)^2}$$

This is equivalent to:

$$\frac{\delta(\beta-\phi)}{1-\delta}\left[\frac{\delta\beta+\delta\phi}{1+\delta}-X\right] > 0.$$

So, if $\beta < \phi$ the inequality is satisfied as long as X is bigger than $(\delta\beta+\delta\phi)/(1+\delta)$ which is obviously less than one. Yet X is bigger than one since by subtracting the denominator of X from its numerator one obtains:

$$2-\delta\phi-\delta\beta+2\delta-\delta^2\phi-\delta^2\beta-[2-(\delta\phi)^2-(\delta\beta)^2] = \delta(1-\phi)(1-\delta\phi) + \delta(1-\beta)(1-\delta\beta) > 0$$

This completes the proof.

To prove the inequality (11) for the case where $1/\mu$ is smaller than one we note that, since β is also less than one, (11) can be written as:

$$\frac{(\delta\beta/\mu)^2-(\delta/\mu)^2}{1+\delta/\mu^2} > (\delta\beta/\mu-\delta/\mu)X'$$

where:

$$X' = \frac{(2-\delta\beta/\mu-\delta/\mu)(1+\delta)}{2-(\delta\beta/\mu)^2-(\delta/\mu)^2}$$

This is equivalent to:

$$(\delta\beta/\mu-\delta/\mu)\left[\frac{\delta/\mu+\delta\beta/\mu}{1+\delta/\mu^2}-X'\right] > 0$$

So, if β is smaller than one the inequality is satisfied as long as X' is bigger than $(\delta/\mu+\delta\beta/\mu)/(1+\delta/\mu^2)$. This latter expression is smaller than one since by subtracting the denominator from the numerator one obtains:

$$(\delta/\mu)(\beta-1)+2\delta/\mu-[1-\delta+\delta+\delta/\mu^2] = (\delta/\mu)(\beta-1)-(1-\delta)-\delta(1-1/\mu)^2$$

Moreover X' is greater than one since, by subtracting its denominator from its numerator one obtains:

$$(2-\delta\beta/\mu-\delta/\mu)(1+\delta)-(2-(\delta B/\mu)^2-(\delta/\mu)^2) = (1-\beta/\mu)(\delta-\delta^2\beta/\mu) + (1-1/\mu)(\delta-\delta^2/\mu) > 0$$

This completes the proof.

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[Footnotes]

[2] **The Kinky Oligopoly Demand Curve and Rigid Prices**
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