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Dynamic Factor Demands under Rational Expectations*

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Abstract

We model the industrial demands for structures, equipment, and blue- and white-collar labor in a manner consistent with rational expectations and stochastic dynamic optimization in the presence of adjustment costs, but allowing generality of functional form. We represent the technology by a translog input requirement function that specifies the amount of blue-collar labor needed to produce an output level given quantities of three quasi-fixed factors: non-production workers, equipment, and structures. A complete description of the production structure is obtained by simultaneously estimating the input requirement function and three stochastic Euler equations. We find adjustment costs are small in total but large on the margin, and differ considerably across factors.

I. Introduction

Understanding the way in which tax changes, changes in relative factor prices, and changes in aggregate output affect investment and employment over time requires a model of the production structure that incorporates dynamic adjustment of "quasi-fixed" factors, i.e. a dynamic model of factor demands. Such a model is developed in this paper in a way that is consistent with rational expectations and dynamic optimization in the presence of adjustment costs, while allowing for generality of functional form.

Dynamic factor demand models are certainly not new to the literature. The "flexible accelerator" and related models of investment demand have been widely used in empirical applications, although they are generally based on *ad hoc* descriptions of the dynamic adjustment process.[1] Berndt, Fuss & Waverman (1980) and Morrison & Berndt (1981) developed dynam-

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[1] As Lucas (1967) and Treadway (1971) have shown, under certain conditions the flexible accelerator is consistent with dynamic optimization in the presence of adjustment costs. For a survey of this and related models, together with an assessment of their empirical performance, see Clark (1979).

ic models in which capital is quasi-fixed and subject to quadratic adjustment costs, but their approach utilizes an explicit solution to the optimal investment problem. In so doing it imposes the assumption that producers have static expectations regarding the evolution of factor and output prices, and requires that the underlying cost function be quadratic. Kennan (1979) and Meese (1980) estimated dynamic factor demand models in which producers have rational expectations, but Kennan imposed the restriction of a linear production structure, and Meese imposed a quadratic production structure. Epstein & Denny (1983) estimate the factor demands implied by a fairly general prespecified value function. However, in their approach, firms act as though they knew their future environment (prices, output, and interest rates) with certainty.[2]

In an earlier paper (1983) concerned primarily with energy demand, we demonstrated an alternative approach that allows for a general production structure and dynamic optimization under uncertainty. It works as follows. In a stochastic environment, firms that have rational expectations and maximize the expected sum of discounted profits also minimize the expected sum of discounted costs. Given any restricted cost function, one can derive the stochastic Euler equations (one for each quasi-fixed factor) that hold for this latter dynamic optimization problem. These Euler equations are just first-order conditions, and although they do not provide a complete solution to the optimization problem, they can be estimated directly for any parametric specification of the technology.

Our approach is to represent the technology by a translog restricted cost function, and then estimate the Euler equations, together with the cost function itself and the static demand equations for any flexible factors, using three-stage least squares. This permits us to test structural restrictions such as constant returns, and to test the over-identifying restrictions implied by rational expectations. The estimated equations then provide a complete empirical description of the production technology, including both short-run (only flexible factors adjust) and long-run (all factors fully adjust) elasticities of demand. The parameter estimates are fully consistent with rational expectations, and in particular with firm behavior that utilizes the solution to the underlying stochastic control problem.[3]

In our earlier paper we used US manufacturing data for the period

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[2] For a survey of some of the recent work in dynamic factor demand modelling, see Berndt, Morrison & Watkins (1981).

[3] Since we do not actually solve the stochastic control problem (beyond writing the first-order conditions), we cannot calculate optimal factor demand trajectories corresponding to particular stochastic processes for prices. Stochastic control problems of this sort are generally difficult, if not impossible to solve, and this raises the question of whether rational expectations provides a realistic behavioral foundation for studying investment behavior and factor demands in general.

1948–1971 to estimate a model in which capital and labor were treated as quasi-fixed factors, and energy and materials as flexible factors. We found adjustment costs on capital to be very important, but adjustment costs on labor appeared negligible. One might think that this latter result was due to our aggregation of white-collar (skilled) and blue-collar (unskilled) labor, and we explore this question here.

In this paper we utilize the same methodological approach described above, but both the model and data are different, and we focus on a number of different issues. Here we ignore the role of energy and materials, but we disaggregate labor (into white-collar and blue-collar) and capital (into equipment and structures). This disaggregation turns out to be quite revealing. We find that adjustment costs for white-collar labor are statistically significant, but quite small in magnitude. We also find that adjustment costs on structures and equipment are quite different, and that structures and equipment enter the production structure differently. In addition, this paper studies the role of financing in more detail, and deals explicitly with the effects of both corporate and personal taxes on factor demands. We allow for both debt and equity financing, and *estimate* the extent to which firms borrow to finance the marginal dollar of investment. Finally, we estimate the model with data for the period 1949–1976, thereby including those more recent years during which factor prices and rates of return fluctuated widely.

Our basic methodological approach is summarized in the next section. There we specify the translog restricted cost function (since blue-collar labor is the only flexible factor this boils down to an input requirement function, and there are no static demand equations), and derive the stochastic Euler equations for the quasi-fixed factors. Section III briefly summarized the estimation method, and discusses the treatment of various taxes and other issues related to the data. Estimation results are presented and discussed in Section IV.

II. The Models

Before presenting the details of our particular model specifications, it is useful to briefly review our general approach to estimating the production structure of a firm facing adjustment costs and making input choices in an uncertain environment. Let us assume that at time τ the firm chooses levels of n variable inputs whose quantities and nominal prices are given by the vectors $\mathbf{V}_\tau=(V_{i\tau})$ and $\mathbf{v}_\tau=(v_{i\tau})$ respectively, and m quasi-fixed inputs whose quantities are given by the vector $\mathbf{X}_\tau=(X_{i\tau})$. These inputs yield the single output Q_τ. The technology can therefore be represented by a restricted cost function C_τ which specifies the minimum expenditure on variable factors needed to produce Q_τ, given the amounts of quasi-fixed factors $\mathbf{X}_\tau$:

$$C_\tau = C(\mathbf{v}_\tau, \mathbf{X}_\tau, Q_\tau, \tau), \tag{1}$$

with C increasing and concave in $\mathbf{v}$ but decreasing and convex in $\mathbf{X}$, and the dependence on τ capturing technical progress.

By definition, the firm incurs costs of adjusting the quasi-fixed factors. We assume these adjustment costs are convex and external to the firm,[4] and we represent them in nominal terms by $P_\tau h(\Delta\mathbf{X}_\tau)$, where P_τ is the price of output and $\Delta\mathbf{X}_\tau = X_\tau - X_{\tau-1}$. The firm also makes direct outlays for its use of quasi-fixed factors, and because of tax and financing considerations these expenditures may be spread through time. We therefore assume that outlays (net of adjustment costs) for quasi-fixed factors at τ, H_τ, are a function of the current and past quantities of those factors:

$$H_\tau = H(\mathbf{X}_\tau, \ldots, \mathbf{X}_{\tau-T}). \tag{2}$$

We assume the firm maximizes its expected present discounted value of profits. As shown in our earlier paper (1983), this implies that the firm minimizes the expected present discounted value of costs. Thus at time t the firm chooses a contingency plan for the vector of quasi-fixed factors to minimize:

$$\min_{\{\mathbf{X}\}} \mathscr{E}_t \sum_{\tau=t}^{\infty} R_{t,\tau}[C_\tau + P_\tau h(\Delta\mathbf{X}_\tau) + H_\tau] \tag{3}$$

where $\mathscr{E}_t$ denotes the expectation conditional on information available at t, and $R_{t,\tau}$ is the discount factor applied at t for costs incurred at τ.[5]

By taking the derivative of (3) with respect to X_{it} and setting it equal to zero, it is easily seen that the minimization yields the following Euler equations, or first-order conditions, for $i=1, \ldots, m$:

$$\mathscr{E}_t\left[\frac{\partial C_t}{\partial X_{it}} + P_t \frac{\partial h(\Delta\mathbf{X}_t)}{\partial X_{it}} - R_{t,t+1} P_{t+1} \frac{\partial h(\Delta\mathbf{X}_{t+1})}{\partial X_{it}} + \sum_{j=0}^{T} R_{t,t+j} \frac{\partial H_{t+j}}{\partial X_{it}}\right] = 0 \tag{4}$$

These Euler equations just state that the net change in expected discounted costs from hiring one more unit of X_i at t is zero. That change is the sum of

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[4] As in Gould (1968). For a survey of the treatment of adjustment costs, see Söderström (1976). We implicitly assume that costs of adjustment are incurred only when the *net* quantity of X_{it} changes. This assumption was relaxed in our earlier paper (1983), where our point estimates suggest that, indeed, only net changes in X_{it} lead to adjustment costs. We also estimated the current models relaxing this requirement, without affecting our results significantly.

[5] Note that while the expectation in (3) treats not just future input prices but also future output as random variables, output is not predetermined. The random variable Q_τ is given by the contingengy plan that maximizes expected profits. For a more detailed discussion of this point, see Pindyck & Rotemberg (1983).

the increase in variable costs (which is negative), the extra costs of adjustment at t, the expected discounted value of the extra expenditures which the firm must incur by holding (at t only) one extra unit of X_i, minus the expected discounted value of the savings in future costs of adjustment. Note that the last term in eqn. (4) is a general expression for what is usually referred to as the rental rate on capital.

By Shepherd's Lemma we have the following additional first-order conditions, which take the form of static demand equations for the flexible factors:

$$V_{it} = \partial C_t / \partial v_{it}, \quad i = 1, \ldots, n. \tag{5}$$

As we discuss in more detail later, our approach is to estimate equations (1), (4), and (5) simultaneously using an instrumental variables procedure.

In this paper we focus on four production inputs: equipment, structures, and white-collar and blue-collar labor. Our model pertains to the US manufacturing sector, which we treat as a single firm[6] that takes input prices as given. Data for other inputs (e.g. energy and raw materials) were not available for the time period we consider, and we assume that such inputs are not substitutable for labor and capital. In particular, we assume that expenditures on blue-collar workers (the only variable input) depend only on the wage of those workers, the levels of the other three inputs, and output.[7]

Since there is only one variable input, the restricted cost function (1) takes the form of an input requirement function, which we specify in translog form:

$$\begin{aligned}\ln B_t = {} & \alpha_0 + \alpha_1 \ln L_t + \alpha_2 \ln E_t + \alpha_3 \ln S_t + \alpha_4 \ln Q_t + \tfrac{1}{2}\gamma_{11} (\ln L_t)^2 \\ & + \gamma_{12} \ln L_t \ln E_t + \gamma_{13} \ln L_t \ln S_t + \gamma_{14} \ln L_t \ln Q_t + \tfrac{1}{2}\gamma_{22} (\ln E_t)^2 \\ & + \gamma_{23} \ln E_t \ln S_t + \gamma_{24} \ln E_t \ln Q_t + \tfrac{1}{2}\gamma_{33} (\ln S_t)^2 + \gamma_{34} \ln S_t \ln Q_t \\ & + \tfrac{1}{2}\gamma_{44} (\ln Q_t)^2 + \lambda t \end{aligned} \tag{6}$$

where B_t, L_t, E_t, S_t, and Q_t are the levels of blue-collar labor, white-collar labor, equipment, structures and output at t. The term λt allows for neutral technical progress. (The restrictions of the α's and γ's required for this translog function to be decreasing and convex in L_t, E_t and S_t are not imposed in the estimation.)

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[6] Or, equivalently, as consisting of many competitive firms whose aggregate technology is given by our model.

[7] This assumption is justified if the production function is of the Leontief form in two composite inputs, the first of which is a function of the levels of labor and capital while the second is a function of the other inputs.

Costs of adjustment are assumed to be quadratic in ΔL_t, ΔE_t, and ΔS_t.[8] In particular,

$$h = h_L(\Delta L_t)+h_E(\Delta E_t)+h_S(\Delta S_t) = \tfrac{1}{2}\beta_L(\Delta L_t)^2+\tfrac{1}{2}\beta_E(\Delta E_t)^2+\tfrac{1}{2}\beta_S(\Delta S_t)^2.$$

Note that cross effects—i.e. changes in one factor affecting costs of adjusting other factors—are neglected. Letting w_t and b_t be the hourly wage to white-collar and blue-collar workers respectively, the Euler equation for white-collar labor is then given by:

$$\begin{aligned}&[b_t(1-\theta_t)B_t/L_t](\alpha_1+\gamma_{11}\ln L_t+\gamma_{12}\ln E_t+\gamma_{13}\ln S_t+\gamma_{14}\ln Q_t)\\&\quad+w_t(1-\theta_t)+P_t\beta_L(L_t-L_{t-1})-\beta_L\,\mathscr{E}_t[R_{t,t+1}P_{t+1}(L_{t+1}-L_t)]=0 \qquad (7)\end{aligned}$$

where θ_t is the corporate income tax rate (wages are tax deductible in the US).

The importance of tax and financing considerations becomes clear when we consider equipment. Let e_t be the purchase price of a unit of equipment. Suppose the firm buys a unit of equipment at t with the intention of keeping it until it is fully depreciated. If the firm borrows the present discounted value of the depreciation allowances its after-tax payment would be $e_t\times(1-c_{Et}-z_{Et})$, where c_{Et} is the investment tax credit on equipment and z_{Et} is the present discounted value of the depreciation allowance. If the firm wants an extra unit of equipment at t without affecting the level of capital in subsequent periods, it will purchase $(1-\delta_E)$ fewer units of equipment at $t+1$, where δ_E is the physical depreciation rate for equipment, thereby saving $(1-\delta_E)\,e_{t+1}(1-c_{E,t+1}-z_{E,t+1})$.[9] So far the only debt the firm incurs is offset by the depreciation allowances. However, we would like to allow for the possibility that a fraction d of $e_t(1-c_{Et}-z_{Et})$ is borrowed for the purchase of the marginal dollar's worth of equipment, and for simplicity we assume (perhaps unrealistically) that this *marginal* debt is repaid after one year. (However, the firm is also allowed to borrow unspecified amounts on *inframarginal* units of equipment.) Given this treatment of taxes and financing, and assuming that factors are productive in the period in which they are purchased, the Euler equation for equipment is:

$$\begin{aligned}&[b_t(1-\theta_t)B_t/E_t](\alpha_2+\gamma_{12}\ln L_t+\gamma_{22}\ln E_t+\gamma_{23}\ln S_t+\gamma_{24}\ln Q_t)\\&\quad+(1-d)\,e_t(1-c_{Et}-z_{Et})+P_t\beta_E(E_t-E_{t-1})\end{aligned}$$

[8] We also estimated versions of the model assuming adjustment costs are quadratic in $\Delta L_t/L_{t-1}$, $\Delta E_t/E_{t-1}$, and $\Delta S_t/S_{t-1}$, without significantly affecting the results.

[9] This assumes the firm purchases some equipment at $t+1$. If the firm bought an extra unit of equipment at t with the intention of *selling* $(1-\delta_E)$ units at $t+1$, it would be unlikely to borrow the expected present value of the depreciation allowances. Instead it would pay $e_t(1-c_{Et}-\tilde{z}_{Et})$ at t, where $\tilde{z}_{Et}$ is the depreciation allowances in the first period, and it would receive $(1-\delta_E)\,e_{t+1}(1-c_{E,t+1})$ at $t+1$.

$$+\mathscr{E}_t R_{t,t+1}\{d[1+i_t(1-\theta_{t+1})]e_t(1-c_{Et}-z_{Et})-(1-\delta_E)\,e_{t+1}(1-c_{E,t+1}-z_{E,t+1})$$
$$-P_{t+1}\beta_E(E_{t+1}-E_t)\}=0 \qquad (8)$$

where i_t is the pre-tax rate of interest paid at $t+1$ on marginal borrowing at t.[10]

A similar analysis for structures yields the following Euler equation for that factor:

$$[b_t(1-\theta_t)\,B_t/S_t](\alpha_3+\gamma_{13}\ln L_t+\gamma_{23}\ln E_t+\gamma_{33}\ln S_t+\gamma_{34}\ln Q_t)$$
$$+(1-d)\,s_t(1-c_{St}-z_{St})+P_t\beta_S(S_t-S_{t-1})+\mathscr{E}_t R_{t,t+1}$$
$$\times\{d[1+i_t(1-\theta_{t+1})]s_t(1-c_{St}-z_{St})-(1-\delta_S)\,s_{t+1}(1-c_{S,t+1}-z_{S,t+1})$$
$$-P_{t+1}\beta_S(S_{t+1}-S_t)\}=0 \qquad (9)$$

where δ_S, s_t, c_{St}, and z_{St} are, respectively, the physical depreciation rate, the purchase price, the investment tax credit, and the present value of depreciation allowances for structures.

The model given by equations (6)—(9) is our "preferred" specification. One might argue, however, that the distinction between equipment and structures is somewhat artificial, since any capital that cannot be easily removed is classified as a structure. We therefore estimate an alternative model for which equipment and structures are aggregated into a single measure of capital. The blue-collar labor input requirement function is again specified to be translog:

$$\ln B_t=\phi_0+\phi_1\ln L_t+\phi_2\ln K_t+\phi_3\ln Q_t+\tfrac{1}{2}\psi_{11}(\ln L_t)^2+\psi_{12}\ln L_t\ln K_t$$
$$+\psi_{13}\ln L_t\ln Q_t+\tfrac{1}{2}\psi_{22}(\ln K_t)^2+\psi_{23}\ln K_t\ln Q_t+\tfrac{1}{2}\psi_{33}(\ln Q_t)^2+\lambda t \qquad (10)$$

where K_t is the quantity of aggregate capital.

Letting δ_K, k_t, c_{Kt}, and z_{Kt} denote, respectively, the physical depreciation rate, purchase price, investment tax credit, and present discounted value of depreciation allowances for this capital, the Euler equations are now given by:

$$[b_t(1-\theta_t)\,B_t/L_t](\phi_1+\psi_{11}\ln L_t+\psi_{12}\ln K_t+\psi_{13}\ln Q_t)+w_t(1-\theta_t)$$
$$+P_t\beta_L(L_t-L_{t-1})-\beta_L\,\mathscr{E}_t[R_{t,t+1}P_{t+1}(L_{t+1}-L_t)]=0 \qquad (11)$$

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[10] If revenues at $t+1$ get discounted at t at the rate $i_t(1-\theta_{t+1})$, then $R_{t,t+1}=1/(1+i_t(1-\theta_{t+1}))$, and the fraction d that is debt financed is irrelevant to the firm. However, it is more reasonable to assume that future costs incurred by firms get discounted at the rate of return on equity. This is the rate at which share-holders, whose objectives the firm represents, discount future revenues and costs.

$$[b_t(1-\theta_t)B_t/K_t](\phi_2+\psi_{12}\ln L_t+\psi_{22}\ln K_t+\psi_{23}\ln Q_t)+(1-d)k_t(1-c_{Kt}-z_{Kt})$$
$$+P_t\beta_K(K_t-K_{t-1})+\mathscr{E}_t R_{t,t+1}\{d[1+i_t(1-\theta_{t+1})]k_t(1-c_{Kt}-z_{Kt})$$
$$-(1-\delta_K)k_{t+1}(1-c_{K,t+1}-z_{K,t+1})-P_{t+1}\beta_K(K_{t+1}-K_t)\}=0 \quad (12)$$

III. Estimation Method and Data

We obtain parameter values for "Model 1" by simultaneously estimating the input requirement function (6) and the Euler equations (7), (8), and (9), and for "Model 2" by simultaneously estimating the input requirement function (10) and the Euler equations (11) and (12). Note that because there is only one variable factor in each model, no static demand equations are estimated.

The estimation is done using three-stage least squares, which, if the error terms are conditionally homoscedastic, is equivalent to the generalized instrumental variables procedure of Hansen (1982) and Hansen & Singleton (1982). That procedure minimizes the correlation between variables known at t (the instruments) and the residuals of the estimating equations. These residuals result from using the actual values of the prices and quantities at time $t+1$ in the Euler equations (7), (8), (9), (11) and (12). The minimized value of the objective function of this procedure provides a statistic J, which is distributed as chi-squared, and which can be used to test the over-identifying restrictions of the model, as well as structural restrictions such as constant returns.

This instrumental variables procedure is a natural one to apply to the Euler equations. This is because the residuals of those equations can be viewed as expectational errors which, if agents have rational expectations, have mean zero conditional on information available at t. The residuals of the input requirement function, however, cannot be interpreted in the same way, since in theory that function should provide an exact description of the technology. In practice that function will not fit the data exactly, and the errors are likely to be correlated with variables known at t. We will assume that this function holds in expectation with respect to some smaller conditioning set (i.e. set of instruments). We take that set to exclude current variables appearing in the input requirement function or Euler equations.

We estimate the models using annual data for the US manufacturing sector for the years 1949–1976. Quantities and wage rates for blue- and white-collar labor are those compiled in Berndt and Morrison (1979).[11] The purchase prices and quantities of equipment and structures were construct-

[11] These data incorporate some embodied technical progress since employment levels are corrected for the educational achievement of the work force.

ed by Ernst Berndt using a perpetual inventory method, and assuming a physical depreciation rate for equipment of 0.135 and for structures of 0.071. (In "Model 2" equipment and structures are aggregated using a Divisia Index). Since no data for gross output in manufacturing is available for the period of our study, we used the level of the gross domestic product of manufacturing from the National Income and Product Accounts. The price index P_t came from the same source.

We assume that the 1-period discount rate $R_{t,t+1}$ is equal to $1/(1+r_e)$, where r_e is the after-tax return on equity.[12] This return is constructed from the identity:

$$r_e = r_d(1-\theta_p)+r_c(1-\theta_c), \tag{13}$$

where r_d and r_c are respectively the dividend yield and capital gains rate (we use data on the Standard and Poor's 500 Index for both), θ_p is the marginal personal tax rate (we use data reported by Seater (1980), and θ_c is the *effective* tax rate on capital gains (based on the estimates of Feldstein & Summers (1979), we set θ_c=0.047). As for the interest rate i_t, we use the rate on commercial paper. The investment tax credit and present discounted value of depreciation allowances are computed by Jorgenson & Sullivan (1981), who use data on the term structure of interest rates to obtain z.

The following instruments are used in the estimation of both models: a constant, and the lagged detrended values of the following variables: rate of return on equity, the hourly compensation of both types of workers, the purchase prices of equipment and structures, the present discounted value of their depreciation allowances, as well as the logarithms of the quantities of blue-collar labor, white-collar labor, structures, equipment, and output.

IV. Estimation Results

For our "prefered" specification, both capital and labor are disaggregated, so that there are three quasi-fixed factors. Recall that the model for this specification ("Model 1") is given by the input requirement function (6) and the three Euler equations (7), (8), and (9). We also estimate the alternative specification in which equipment and structures are aggregated; this model ("Model 2") is given by the input requirement function (10) and the Euler equations (11) and (12). Each of these models was estimated first in its unrestricted form, then with the restrictions of homotheticity imposed, and finally with the restrictions of constant returns (CRTS).[13]

[12] We also estimated the models using $R_{t,t+1}=1/(1+r_{cp})$, where r_{cp} is the after-tax return on commercial paper, but we obtained significantly poorer results.

[13] Note that the homothetic and constant returns versions are estimated *using the covariance matrix obtained from the estimation of the unrestricted model,* thereby permitting us to test these restrictions.

Table 1. *Parameter estimates for Model 1 (B; L, S, E)*
Asymptotic standard errors in parentheses

	Unrestricted	Homothetic	CRTS
α_0	−12.1166	−3.7658	7.8492
	(2.6060)	(1.1064)	(1.2135)
α_1	4.4941	3.9380	−0.7089
	(0.3352)	(0.3413)	(0.6326)
α_2	0.00075	−0.003389	0.001997
	(0.00090)	(0.00132)	(0.000541)
α_3	−0.000677	0.000141	0.001844
	(0.00082)	(0.00106)	(0.000652)
α_4	0.5907		
	(0.9379)		
γ_{11}	−0.7502	−0.6492	0.03281
	(0.0614)	(0.0608)	(0.1642)
γ_{12}	3.010×10^{-5}	0.000486	0.000513
	(0.00016)	(0.000213)	(0.000123)
γ_{13}	−0.000103	0.000175	0.000352
	(0.000138)	(0.000159)	(0.000142)
γ_{14}	0.3416		
	(0.04815)		
γ_{22}	0.000191	−0.000050	0.000289
	(0.000129)	(0.00013)	(0.000113)
γ_{23}	−0.000253	−0.000007	−0.000164
	(9.624×10^{-5})	(0.000071)	(7.878×10^{-5})
γ_{24}	−0.000167		
	(0.000194)		
γ_{33}	0.000563	−0.000718	−0.000261
	(0.000277)	(0.000374)	(0.000104)
γ_{34}	6.181×10^{-5}		
	(0.000173)		
γ_{44}	−0.5045		
	(0.1782)		
λ	−0.01295	−0.000059	−0.02890
	(0.00302)	(0.0023)	(0.001186)
d	1.2733	1.2430	1.1938
	(0.09859)	(0.1012)	(0.09526)
β_L	0.000318	0.000318	0.000191
	(5.243×10^{-5}	(0.00005)	(0.000189)
β_E	0.00476	0.00538	0.003159
	(0.00238)	(0.00384)	(0.002264)
β_S	0.009238	0.02547	0.2021
	(0.009418)	(0.01264)	(0.00868)
θ		0.42446	
		(0.03682)	
J	54.45	141.45	418.22
EQ 6 SSR	0.02186	0.05364	0.06721
D.W.	1.181	0.773	0.737
EQ 7 SSR	0.17229	0.19518	3.2188
D.W.	1.727	1.769	0.218
EQ 8 SSR	0.01858	0.0530	0.03566
D.W.	1.766	0.685	0.912
EQ 9 SSR	0.01272	0.02103	0.02079
D.W.	2.771	1.797	1.715

Table 2. *Parameter estimates for Model 2 (B; L, K)*
Asymptotic standard errors in parentheses

	Unrestricted	Homothetic	CRTS
ϕ_0	−10.9216 (2.3371)	−4.782 (1.021)	7.7565 (1.3809)
ϕ_1	4.5063 (0.3342)	4.362 (0.331)	−0.7034 (0.7172)
ϕ_2	−0.003493 (0.003411)	−0.00134 (0.00303)	−0.01004 (0.001362)
ϕ_3	0.1395 (0.8566)		
ψ_{11}	−0.7527 (0.06122)	−0.7611 (0.0595)	0.03160 (0.1863)
ψ_{12}	0.000709 (0.000611)	0.00063 (0.00049)	0.002714 (0.000347)
ψ_{13}	0.3473 (0.04810)		
ψ_{22}	0.000885 (0.000656)	−0.00124 (0.00044)	0.001593 (0.00046)
ψ_{23}	−0.001416 (0.000659)		
ψ_{33}	−0.4247 (0.1620)		
λ	−0.01213 (0.002614)	−0.00200 (0.00165)	−0.02427 (0.000721)
d	1.5977 (0.1752)	1.5655 (0.1610)	1.4381 (0.1635)
β_L	0.000315 (0.000052)	0.000309 (0.000053)	0.000185 (0.000223)
β_K	0.01768 (0.003304)	0.02301 (0.00370)	0.01047 (0.002753)
θ		0.51087 (0.02569)	
J	34.81	61.39	447.83
EQ 10 SSR	0.02154	0.0353	0.09893
D.W.	1.160	0.901	0.506
EQ 11 SSR	0.1717	0.1834	3.2225
D.W.	1.708	1.487	0.217
EQ 12 SSR	0.08443	0.1059	0.2643
D.W.	1.647	1.842	0.568

For Model 1, the parameter restrictions implied by homotheticity are: $\alpha_4=\theta(1-\alpha_1-\alpha_2-\alpha_3)$, $\gamma_{14}=-\theta(\gamma_{11}+\gamma_{12}+\gamma_{13})$, $\gamma_{24}=-\theta(\gamma_{12}+\gamma_{22}+\gamma_{23})$, $\gamma_{34}=-\theta(\gamma_{13}+\gamma_{23}+\gamma_{33})$, and $\gamma_{44}=-\theta(\gamma_{14}+\gamma_{24}+\gamma_{34})$. These restrictions imply that when output increases by 1 percent all factor inputs will increase by θ

percent in long-run equilibrium. For Model 2, the parameter restrictions implied by homotheticity are: $\phi_3=\theta(1-\phi_1-\phi_2)$, $\psi_{13}=-\theta(\psi_{11}+\psi_{12})$, $\psi_{23}=-\theta(\psi_{12}+\psi_{22})$, and $\psi_{33}=\theta(\psi_{13}+\psi_{23})$. Finally, for both models constant returns to scale requires that in addition $\theta=1$.

Parameter estimates are shown in Table 1 for Model 1, and in Table 2 for Model 2. Both models satisfy the condition of monotonicity and convexity for all but the first four years of data. Observe that for the unrestricted version of Model 1 the value of J is 54.45. Under the null hypothesis that the model is valid, J is distributed as chi-square with number of degrees of freedom equal to the number of instruments (13) times the number of equations (4) minus the number of parameters (20). The critical 5% level of the chi-square distribution with 32 degrees of freedom is 46.2, so that the over-identifying restrictions are rejected, throwing some doubt on the validity of the estimated standard errors.[14] For Model 2 the value of J is 34.81. For this model there are 39–14=25 degrees of freedom, the critical 5% level of the chi-square distribution is 37.7, and the over-identifying restrictions can be accepted.

To test for homotheticity, we use the difference in the values of J with and without the parameter restrictions imposed. That difference is 87.0 for Model 1, and 26.3 for Model 2. These numbers are well above the critical 5 percent values of the chi-square distribution with four and three degrees of freedom respectively, so that homotheticity is overwhelmingly rejected. The estimates for θ are near 0.5 for both models, suggesting important economies of scale.

The fact that θ is significantly different from 1 constitutes a rejection of CRTS under the maintained hypothesis of homotheticity. We also test CRTS directly by comparing the J statistics for CRTS imposed with those of the unrestricted models. The differences in the values of J are 363.8 for Model 1, and 413.0 for Model 2, which are much larger than the critical 5 percent chi-square values with five and four degrees of freedom, again rejecting CRTS.[15]

Our unrestricted estimates allow us to compute how much output would increase if all inputs were increased by 1%. For the 1976 data point, Model 1 shows a 1.66% increase in output, and Model 2 shows a 1.64% increase. We thus find increasing returns to scale that are large but not unreasonable.

The parameter estimates have interesting implications for the role of

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[14] Failure of the over-identifying restrictions in this model is inconsistent with the hypothesis that firms are optimizing with rational expectations. It could imply the existence of systematic optimizing or forecasting errors on the part of firms, or a mis-specification of our input requirement function.

[15] Note that this puts into question empirical q-theory models of investment that equate marginal and average q.

Table 3. *Adjustment costs*

	Percentage marginal adjustment cost (avg.)[a]	Percentage total adjustment cost (avg.)[a]
Model 1		
L	0.03	0.001
E	0.23	0.007
S	0.34	0.005
Model 2		
L	0.03	0.001
K	2.14	0.056

[a] Computations are explained in the text.

adjustment costs. Observe that all of the adjustment cost parameters have the correct sign, and all are statistically significant except the one for structures in Model 1 (although this parameter is numerically large). The importance of adjustment costs is best understood by comparing their value as a total fraction of expenditures on a particular quasi-fixed factor with their value on the margin. This is done in Table 3. In the first column we take the average annual change in the stock of each quasi-fixed factor over the sample period, compute the marginal adjustment cost for that average

Table 4. *Elasticities*

Model 1 (B; L, S, E)					
Long-run elasticity of demand for	With respect to				
	P_B	P_L	P_E	P_S	Q
B	−1.4505	1.4505	-1.493×10^{-5}	1.734×10^{-5}	0.3575
L	2.3105	−2.3106	0.000126	1.0007×10^{-5}	0.9938
E	−0.1554	0.8166	−0.5221	−0.1390	1.0582
S	0.2708	0.09809	−0.2107	−0.1582	0.5282
Short-run elasticity of demand for		With respect to			
		L	E	S	Q
B		−0.6278	-9.7065×10^{-5}	-6.4045×10^{-5}	0.98158

Model 2 (B; L, K)				
Long-run elasticity of demand for	With respect to			
	P_B	P_L	P_K	Q
B	−1.4674	1.4676	−0.000229	0.3513
L	2.3381	−2.3385	0.000394	1.0187
K	−1.6851	1.8186	−0.1335	0.7275
Short-run elasticity of demand for		With respect to		
		L	K	Q
B		−0.62769	−0.000136	0.99086

annual change using the parameter estimates of Tables 1 and 2, and then divide by the average rental rate for the factor. This provides a measure of percentage marginal adjustment costs. In the second column we compute total adjustment costs for each factor in each year as a fraction of the total expenditure on that factor, and then average these figures over the sample period. This provides a measure of percentage total adjustment costs. Observe that while adjustment costs are small as a total percentage of expenditures, they are quite large on the margin for capital. This means that the firm's cost minimization problem is very much a dynamic one. However, adjustment costs for white-collar labor, both on the margin and as a percentage of total cost, are small. We also found very small adjustment costs for labor in our earlier paper (1983), in which labor is aggregated.

Our estimates also have implications for the role of financing. In particular the parameter d specifies the fraction of the after-tax cost of a dollar of capital that is debt financed at the margin. However, our estimates of d are implausibly high since they are significantly greater than one for both models. While the parameter d applies only at the margin and thus the inframarginal investments may well lead to lower borrowing, these values seem high nonetheless.

Further insight into the production structure can be obtained by examining the elasticities of factor demands.[16] These elasticities were calculated for the 1976 sample point, and are presented in Table 4.[17]

A number of things should be mentioned about these elasticities. First, note that there is consistency across the two models. Elasticities of blue- and white-collar labor demand with respect to the two wage rates and output are almost identical across the two models, and in Model 2, the elasticities of demand for capital with respect to its own price and output are about midway between the corresponding elasticities for equipment and structures in Model 1.

Second, note that the elasticities are generally reasonable, both in magnitude and in sign. The short-run elasticities with respect to other factor inputs are negative in both models, as required by monotonicity. In Model 1, blue-collar labor and equipment are complementary inputs in the long run, as are structures and equipment, and in Model 2 blue-collar labor and

[16] Intermediate- and long-run elasticities, i.e. those that apply when quasi-fixed factors have partially or fully adjusted, must be interpreted with caution. The reason is that if prices evolve stochastically, the adjustment path for any particular discrete change in a price, as well as the long-run expected equilibrium, are solutions to a stochastic control problem (and in some cases a long-run expected equilibrium may not exist). Since it is typically infeasible to obtain such solutions, we must compute elasticities by implicitly assuming that firms ignore the variance of future prices in responding to price changes. These elasticities can therefore be best viewed as a description of the technology.

[17] The formulas used to calculate these elasticities are available from the authors on request.

the capital aggregate are complements in the long run. As one would expect, structures is relatively price inelastic, and in particular less elastic than equipment. The small own-price elasticity for the capital aggregate in Model 2 is disturbing by itself, and also when contrasted with the much larger values found in our earlier study (1983). The elasticities of demand for white-collar labor are large, but still reasonable.[18]

Although we do not do so here, one could use the models presented in this paper to simulate the effects of changes in factor prices or output—but only in a deterministic context. As explained in Pindyck and Rotemberg (1983), such simulations are carried out by numerically solving the deterministic control problem that corresponds to the minimization in eqn. (3). A solution to the control problem can be obtained by finding initial conditions for the quasi-fixed factors which yield steady-state values for those factors (i.e. values which satisfy the associated transversality conditions) when the Euler equations and input requirement function are together solved recursively through time. Note that for Model 1, in which there are three quasi-fixed factors, this involves searching over a three-dimensional grid of initial conditions.

V. Conclusions

The model of factor demands presented in this paper is consistent with rational expectations and dynamic optimization in the presence of adjustment costs. With the possible exception of the parameter which describes the manufacturing sector's financing decisions, our estimates are quite plausible. We obtain reasonable elasticity estimates, and find that equipment is a complementary factor to both blue-collar labor and structures, while other factor pairs are substitutes. As in our previous paper, we strongly reject constant returns to scale. Finally, we find that adjustment costs are important at the margin, especially for equipment and structures.

It is important to keep in mind that our model has a number of limitations, some of which are suggestive of further work. First, we aggregate across goods, factors, and time. The numerous outputs produced under the heading of manufacturing are treated as a single good. Factor diversity is in fact richer than the two types of labor and capital we consider. Also, our use of annual data requires the implicit assumption that seasonal fluctuations are such that average output is only a function of average factor use over the year.

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[18] A surprising result in both models is that the elasticity of each type of labor with respect to its own price is nearly the negative of the elasticity with respect to the price of the other type. This is a result of the coincidental fact that for the later data points, the labor price ratios are nearly equal to the ratios of the slopes of the demands.

Second, our model imposes constraints on firms' financial policies. Although we allow for the different tax consequences of debt and equity financing, we ignore the term structures of the debt that firms incur when they purchase new capital. Finally, we only allow for limited forms of technical progress. Expanding the model to include other forms of technical progress might significantly affect the parameter estimates.

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