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Exchange-Rate Dynamics With Sticky Prices: The Deutsche Mark, 1974–1982

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This article estimates simultaneously dynamic equations for the Deutsche mark/dollar exchange rate and the German wholesale price index. These emerge from a model in which German prices are sticky. The stickiness is due to costs of adjusting prices of the form posited by Rotemberg (1982a,b). The main results of the empirical analysis are first that the version of the model in which prices are perfectly flexible is rejected and, second that, in spite of the substantial price stickiness, we find that the nominal exchange rate undershoots in response to monetary innovations like those that appear to be typical in Germany.

KEY WORDS: Costs of price adjustment; Overshooting; Rational expectations.

1. INTRODUCTION

The idea that purchasing-power parity holds with flexible exchange rates at every moment in time has been shown to be wrong by the data since 1974. Real exchange rates—that is, the ratios of aggregate price levels expressed in a common numeraire—are not constant, and, moreover, their changes tend to persist (see Frenkel 1981). Furthermore, nominal exchange rates fluctuate more than price levels. One possible explanation for these phenomena, advanced by Dornbusch (1976), is that prices of goods produced for the domestic market change slowly. If asset markets are always in equilibrium and goods prices move slowly, most of the volatility of the real exchange rate will be accounted for by movements of the nominal exchange rate. Our article attempts to estimate a model of this type on German data since 1974. We too assume that prices move slowly. On the other hand, this article differs in important respects both from Dornbusch's original formulation and from its empirical implementation by Frankel (1979), Driskill and Sheffrin (1981), Backus (1984), Papell (1984), and Barr (1984).

First, we consider the impact of a relatively broad set of forcing variables on the paths of the price level and the exchange rate. In particular, we specify and estimate the effects of changes in prices of various imported goods and of changes in real wages. The effect of these latter variables on exchange rates was the focus of the theoretical work of Sachs (1980), among others.

Second, our price adjustment rule is not exogenously imposed. It arises instead from an explicit optimization problem followed by firms that, as in Rotemberg (1982a), are concerned that customers will desert them if they follow an erratic price behavior. This specification allows for some contemporaneous reaction of the price level to current exogenous shocks, unlike the model estimated by Papell (1984) and Barr (1984) in which quarterly price levels are predetermined.

Finally, we do not impose the assumption that forcing variables follow a random walk. Driskill and Sheffrin (1981) rejected this hypothesis for the German money stock. Thus in our model the responses of exchange rates and prices to innovations in the forcing variables differ from those studied in the theoretical model of Dornbusch (1976), as well as from the simulation of Backus (1984). We let the data inform us as to the plausible stochastic processes followed by the forcing variables that we consider. Then we assume that exchange rates and prices respond optimally in that private agents exploit their knowledge of these stochastic processes.

While we feel that these differences constitute important improvements over previous work, we must point out at the outset some important limitations. First, the period since 1974 is only partially a period of "flexible" exchange rates between Germany and the rest of the world, which we aggregate into a dollar region. This is so because within Europe exchange rates are only allowed to move within bands. So whether flexible-exchange-rate models can explain the Deutsche mark (DM) exchange rate is an empirical question. Second, our model fails to incorporate the effects of government spending, the dynamics due to capital accumulation, and more generally the consequences of varying real interest rates. These simplifications allow us to derive closed-form expressions for endogenous variables, which we use to obtain maximum likelihood estimates of the parameters and to perform simulations.

The structure of the article is as follows. Section 2 presents the model. Section 3 estimates the model using German data for the period from June 1974 to February 1983. Section 4 presents simulations of the responses of the endogenous variables to a variety of shocks and assesses the out-of-sample performance of our model. Finally, Section 5 contains some concluding remarks. The appendix describes the solution of the model and the data used in the empirical section.

2. THE MODEL

Our economy is populated by a large number of monopolistically competitive firms, each producing a good that is differentiated from other domestic goods and from foreign goods. Each firm faces the following demand curve for its product:

$$Q_{it} = \left(\frac{P_{it}}{P_{dt}^{\lambda}(E_t P_{ct}^*)^{1-\lambda}}\right)^{-\gamma} \times \left(\frac{M_t}{P_{dt}^{\lambda}(E_t P_{ct}^*)^{1-\lambda}}\right)^{d} Q_t^{*f} N_{1t}, \quad (1)$$

where Q_{it} is demand for good i at time t. The first term on the right side of (1) represents substitution between domestic and foreign goods and captures substitution both by domestic residents and by foreigners. P_{dt} is the index of domestic goods prices, a geometrically weighted average of P_{it}'s. P_{ct}^* is the price of foreign consumption goods, taken as exogenous. This implies that, although our country is "small," each domestic producer does not face a perfectly elastic demand schedule by foreign residents. The second term on the right side of (1) represents a real balance effect on domestic demand; Q_t^* stands for foreign activity, and N_{1t} is a random variable that affects the demand for goods.

On the cost side, we assume that all domestic goods, together with imported intermediates and labor, are used in the production of each good i. Ignoring constant terms, the marginal cost of producing good i is

$$Q_{it}^{\beta} P_{dt}^{(1-\alpha_1-\alpha_2)} W_t^{\alpha_1} (E_t P_{Nt}^*)^{\alpha_2} N_{2t}, \quad (2)$$

with W_t the nominal wage rate, P_{Nt}^* the foreign currency price of imported intermediate goods, and N_{2t} a random variable affecting productivity. When $\beta = 0$, we have the constant-variable-costs case.

Real wages relative to the consumer price index (CPI) are given by

$$W_t = K_t P_{dt}^{\lambda}(E_t P_{ct}^*)^{1-\lambda}, \quad (3)$$

where K_t is the real consumption wage at time t.

Domestic producers are assumed to observe M, P_c^*, P_N^*, Q^*, E, and K at the time of their pricing decision. In the absence of costs of changing prices, domestic producers would charge $\overline{P}_{it}$, at which marginal cost equals marginal revenue. In natural logarithms, it is equal to

$$\begin{aligned}\overline{p}_{it} = \frac{1}{1+\beta\gamma} \{ & [\beta\lambda(\gamma - d) + 1 - \alpha_1 - \alpha_2 - \lambda\alpha_1] p_{dt} \\ & + [\beta(1-\lambda)(\gamma - d) + (1-\lambda)\alpha_1 + \alpha_2] e_t \\ & + [(\beta(1-\lambda)(\gamma - d) + (1-\lambda)\alpha_1]_{p_{ct}}^{*} \\ & + \alpha_2 p_{Nt}^* + \alpha_1 k_t + \beta\, dm_t + \beta f q_t^* + n_t\}, \end{aligned} \quad (4)$$

where lowercase letters are the logs of the variables represented by the corresponding uppercase letters and $n_t = n_{2t} + \beta n_{1t}$.

We follow Rotemberg (1982a), however, by assuming that monopolists also have convex costs of changing nominal prices. Once a second-order approximation to their profit function is used, the monopolists objective function is

$$\min E_t \sum_{j=0}^{\infty} \rho^j [(p_{it+j} - \overline{p}_{it+j})^2 + c(p_{it+j} - p_{it+j-1})^2], \quad (5)$$

where ρ stands for a constant discount factor, E_t is the expectations operator conditional on information available at time t, and c represents the cost of changing prices.

The first-order conditions for this problem are

$${}_t p_{it+1} - \left(\frac{1 - c + \rho c}{\rho c}\right) p_{it} + \frac{1}{\rho} p_{it-1} = -\frac{1}{\rho c} \overline{p}_{it}, \quad (6)$$

where, for every variable x, ${}_t x_{t+j}$ indicates the expectation of x at time $t + j$ conditional on information available at t. The transversality condition is

$$\lim_{k \to \infty} ({}_t p_{it+k} - {}_t \overline{p}_{it+k}) + c({}_t p_{it+k} - {}_t p_{it+k-1}) = 0. \quad (7)$$

Aggregating Equation (6), one obtains

$$\begin{aligned} {}_t p_{dt+1} - \frac{1}{\rho\phi_3} p_{dt} + \frac{1}{\rho} p_{dt-1} = -\frac{1}{\rho\phi_3} \Big\{ & (1 - (1+\rho)\phi_3 - \phi_7) e_t \\ & + \left(1 - (1+\rho)\phi_3 - \phi_7 - \frac{\alpha_2}{\phi_2}\right) p_{ct}^* \\ & + \frac{\alpha_1}{\phi_2} k_t + \frac{\alpha_2}{\phi_2} p_{Nt}^* + \phi_7 m_t + \phi_6 q_t^* + n_t \Big\}, \end{aligned} \quad (8)$$

where $\phi_2 = (1+\rho)c + \beta\gamma[(1+\rho)c + (1-\lambda)] + \lambda\beta d + (1-\lambda)\alpha_1 + \alpha_2$, $\phi_3 = c(1+\beta\gamma)/\phi_2$, $\phi_6 = \beta f/\phi_2$, and $\phi_7 = \beta d/\phi_2$.

As shown in Equation (16), the solution to (8) that satisfies the aggregate version of the transversality condition (7) is that prices at t depend on prices at $t - 1$ and on the current and expected future values of all of the forcing variables in the model. Iff all of these variables obey a random walk, the solution reduces to the partial adjustment formula of Dornbusch (1976), in which the rate of change of prices is proportional to the difference between $\bar{p}_t$ and p_{t-1}. It is this restrictiveness of the partial-adjustment formula that makes its direct empirical application unappealing. Although numerous theoretical alternatives to partial adjustment have been offered [see Obstfeld and Rogoff (1984) for a survey], the empirical literature (including Buiter and Miller 1982; Driskill and Sheffrin 1981; Frankel 1979; Papell 1984), with the exception of Backus (1984), has focused on equations that do not flow from well-posed individual-optimization problems.

Note two things at this point. First, although Equation (8) is derived under the assumption that there are convex costs of changing prices, the same equation could have been derived from the model of Calvo (1983). This model assumes that producers who discount future profits by ρ are only capable of changing prices in any one period with probability π. This equivalence in the aggregate implications of the two models was proved by Rotemberg (1987). Second, our focus here is on price rigidity only because in Germany overlapping wage contracts of the type observed in the United States do not exist (see Sachs 1979).

The model is closed with the specification of asset-markets equilibrium. The ex ante interest rate differential is specified as follows:

$$ {}_te_{t+1} - e_t = i_t - i_t^* + n_{3t}, \tag{9} $$

where i_t and i_t^* are the domestic and foreign interest rate, respectively, whereas n_{3t} is an exogenous risk premium. Finally, we have the money-demand equation

$$ m_t - \lambda p_{dt} - (1 - \lambda)(e_t + p_{ct}^*) = aq_t - bi_t + n_{4t}, \tag{10} $$

where n_{4t} is a random variable affecting velocity. To obtain the dynamics of the exchange rate, we need to specify the behavior of equilibrium output q_t. We assume that domestic firms are never rationed in the amount of labor and intermediate goods they can buy. They supply whatever quantity is demanded of the good they produce. Then equilibrium domestic output is given by aggregating (1) as follows:

$$ \begin{aligned} q_t &= (e_t + p_{ct}^*)(\gamma - d)(1 - \lambda) \\ &\quad - [\gamma - (\gamma - d)\lambda]p_{dt} + fq_t^* + dm_t + n_{1t}. \end{aligned} \tag{11} $$

The dynamics of the exchange rate are obtained by substituting (11) into money demand and using the relation (9) as follows:

$$ \begin{aligned} {}_te_{t+1} &- \phi_1 e_t - (1 - \phi_1 - \phi_4)p_{dt} \\ &= \phi_4 m_t + (\phi_1 - 1)p_{ct}^* + \phi_5 q_t^* - i_t^* + \tilde{n}_t, \end{aligned} \tag{12} $$

where

$$ \tilde{n}_t = an_{1t}/b + n_{3t} + n_{4t}/b, \qquad \phi_4 = (ad - 1)/b, $$

$$ \phi_1 = 1 + \left(\frac{1 - \lambda}{b}\right)(1 + a(\gamma - d)), $$

and

$$ \phi_5 = af/b. $$

This completes the specification of the model that now consists of the two dynamic equations (8) and (12). For these equations to have a unique solution, we need three boundary conditions. One is given by the predetermined value of p_{dt-1}, and the other is given by the aggregation of (7) across firms. We obtain a third one by assuming that there are no exchange-rate bubbles (so that exchange rates are expected to converge to their long-run value). Since these last two conditions imply only that the system converges over time, they are sufficient to generate a unique solution only if the characteristic equations of the system, (8) and (12), have one stable root and two unstable roots. Then the solution can be written as

$$ \begin{aligned} p_{dt} &= \omega_0 p_{dt-1} - \left(\frac{1}{J_{21}}\right) \sum_{i=1}^{8} \omega_{i1} \left[\sum_{j=0}^{\infty} \left(\frac{1}{J_{21}}\right)^j {}_t z_{it+j}\right] \\ &\quad - \left(\frac{1}{J_{22}}\right) \sum_{i=1}^{8} \omega_{i2} \left[\sum_{j=0}^{\infty} \left(\frac{1}{J_{22}}\right)^j {}_t z_{it+j}\right] \end{aligned} \tag{13} $$

and

$$ \begin{aligned} e_t &= \xi_0 p_{dt-1} - \left(\frac{1}{J_{22}}\right) \sum_{i=1}^{8} \xi_{i1} \left[\sum_{j=0}^{\infty} \left(\frac{1}{J_{22}}\right)^j {}_t z_{it+j}\right] \\ &\quad - \left(\frac{1}{J_{22}}\right) \sum_{i=1}^{8} \xi_{i2} \left[\sum_{j=0}^{\infty} \left(\frac{1}{J_{22}}\right)^j {}_t z_{it+j}\right], \end{aligned} \tag{14} $$

where the ω's, the ξ's, J_{21}, and J_{22} are functions of the parameters of (8) and (12) and the elements of the vector z are m, p_c^*, p_N^*, k, q^*, i^*, n, and $\tilde{n}$.

Although the model exhibits short-run nominal rigidities in that prices and exchange rates depend on lagged prices, it is neutral in the long run with respect to once-and-for-all changes in the stock of money. These eventually lead to equiproportionate increases of p_d and e. The model is not superneutral, however; permanent changes in the rate of growth of nominal variables have real effects.

3. ESTIMATION

The estimation of the structural parameters (the α's and the ϕ's) proceeds in two steps. We first obtain consistent (but inefficient) estimates by instrumental variables applied to (8) and (12). Then, we obtain efficient parameter estimates by maximum likelihood applied to (13) and (14). Maximum likelihood requires that these equations be estimated jointly with the stochastic process of the forcing variables. In practice, the results of these two procedures turn out to be similar.

There are two differences between (8) and (12) on the one hand and the equations we actually estimate by instrumental variables on the other. First, we multiply Equation (8) by ϕ_3. This change in normalization facilitates testing for the absence of costs of changing prices; in that case, $\phi_3 = 0$. Second, we replace expected values at t (${}_tp_{t+1}$ and ${}_te_{t+1}$) with the corresponding realized values. This leads to composite disturbance terms equal to ζ_{1t} for (8) and ζ_{2t} for (12), $\zeta_{1t} = \varepsilon_{1t+1} + n_t$ and $\zeta_{2t} = \varepsilon_{2t+1} + \tilde{n}_t$.

Here ε_{1t+1} is $p_{dt+1} - {}_tp_{dt1}$ and ε_{2t+1} is $e_{t+1} - {}_te_{t+1}$. As Cumby, Huizinga, and Obstfeld (1983) pointed out, ζ_{1t} and ζ_{2t} are likely to be serially correlated. The structural disturbances n_t and $\tilde{n}_t$ affect p_{dt} and e_t unexpectedly, thus changing ε_{1t} and ε_{2t}. Therefore, even when n_t and $\tilde{n}_t$ are not serially correlated, ζ_{1t} and ζ_{2t} are first-order moving average (MA) processes. The estimates of the parameters are obtained taking this into account, since we follow the generalized instrumental variables procedure of Hansen (1982), Hansen and Singleton (1982), and Cumby et al. (1983). The results are presented in Table 1.

Table 1. Generalized Instrumental Variables Estimates of the Price-Level and Exchange-Rate Euler Equations

	1	2	3	4
ϕ_1	1.541	1.567	1.901	1.806
	(.072)	(.067)	(.160)	(.121)
ϕ_3	.497	.497	.501	.501
	(.0023)	(.0021)	(.005)	(.004)
ϕ_4	−.315	−.177	−.117	−.268
	(.095)	(.074)	(.158)	(.179)
ϕ_5	.378	.325	.378	.416
	(.043)	(.034)	(.088)	(.093)
ϕ_6	.005	−.0023	−.006	−.001
	(.003)	(.0011)	(.006)	(.003)
ϕ_7	.003	.002	.003	.002
	(.003)	(.003)	(.007)	(.006)
α_1/ϕ_2	−.064	—	.307	—
	(.028)		(.043)	
α_2/ϕ_2	.002	—	.003	—
	(.001)		(.002)	
ϕ_2	—	53.416	—	39.498
		(26.354)		(29.073)
ρ_1	—	—	.009	.008
			(.424)	(.330)
ρ_2	—	—	.621	.650
			(.159)	(.163)
DW*				
ζ_{1t}, η_{1t}	2.68	2.80	2.88	2.84
ζ_{2t}, η_{2t}	.96	1.04	2.20	2.26
Roots	1.555	1.590	1.891	1.797
	1.116	1.082	1.083	1.058
	.891	.914	.932	.952
J	17.459	26.572	6.260	6.21

NOTE: The sample is for June 1974–January 1983. Standard errors are in parentheses.
* Columns 1 and 2 contain the Durbin–Watson (DW) statistics for the estimates of ζ_{1t} and ζ_{2t}. Columns 3 and 4 contain the DW statistics for the estimates of η_{1t} and η_{2t}.

Column 1 contains estimates for our original specification. Since α_1 and α_2 pertain to production functions estimated more precisely elsewhere, however, we reestimate our model equating them to .25 and .11, respectively. These are the values obtained by Dramais (1980). In addition to identifying ϕ_2, this imposes an additional constraint. Column 2 presents the resulting estimates. The estimates of Columns 1 and 2 are obtained, allowing the disturbance terms to be conditionally heteroscedastic. The instruments are p_d, e, p_c^*, p_N^*, q^*, Δi^*, k, and Δm at $t - 1$. These are valid instruments as long as they are not correlated with either n_t or $\tilde{n}_t$, since, by the rational-expectations assumption, they cannot be correlated with ε_{1t+1} or ε_{2t+1}. We use rates of change rather than levels of m and i^* because, as we shall argue, the estimated stochastic processes of these two variables appear to be stationary only in the first differences.

Columns 1 and 2 show evidence of misspecification. Hansen's (1982) test of overidentifying restrictions, which is reported in the last row, resoundingly rejects these restrictions. The Durbin–Watson (DW) statistic of the exchange-rate equation indicates that the estimated first-order autocorrelation of the residuals is fairly high. This is not inconsistent with the theoretical model because, as argued previously, we expect the composite disturbances to follow a first-order MA process. A high autocorrelation of ζ_{1t} and ζ_{2t} might also be caused by autocorrelation in the structural disturbances n_t and $\tilde{n}_t$, however.

We thus also consider the hypothesis that the structural disturbances follow an autoregressive process:

$$n_t = \rho_1 n_{t-1} + u_{1t}, \qquad \tilde{n}_t = \rho_2 \tilde{n}_{t-1} + u_{2t}. \quad (15)$$

By quasi first-differencing Equations (8) and (12), we obtain estimating equations whose residuals are given by $\eta_{1t} = \varepsilon_{1t+1} - \rho_1\varepsilon_{1t} + u_{1t}$ and $\eta_{2t} = \varepsilon_{2t+1} - \rho_2\varepsilon_{2t} + u_{2t}$. These quasi first-differenced equations are estimated using the same instrumental-variables procedure, with instruments dated at $t - 2$. Once again the errors have an MA component. The results are reported in columns 3 and 4 of Table 1. The estimated autocorrelation coefficient is positive and significant in the exchange-rate equation but insignificant in the price equation. Moreover, the J statistic, which is distributed as a chi-square with 6 df in column 3 and 7 df in column 4, is well within the acceptance range. The table also shows that the estimates are not substantially affected either by quasi first-differencing or by constraining the α's.

To obtain maximum likelihood estimates, we need to express the right sides of Equations (13) and (14) in terms of contemporaneous and past forcing variables. Exchange rates and prices depend on expected future values of such variables. The maximum likelihood procedure imposes the constraint that the stochastic process followed by the forcing variables is known to the public and is used to form the expectations in (13) and

(14). As shown by Hansen and Sargent (1980), the sums of discounted values of the forcing variables can be reduced to a function of present and past values of these variables. The complexity of this function, however, depends on the complexity of the stochastic process followed by the forcing variables. Therefore, we seek to model this stochastic process parsimoniously.

Hansen and Sargent's (1980) formulas require that past values of p_d and e do not help predict the forcing variables. Granger causality tests reported in Table 2 are consistent with this requirement. This table is obtained by first-differencing m and i^* (as the coefficient estimates of a nondifferenced system suggested) and by using six lags for all variables. Lack of Granger causality is related to identification. If past structural disturbances in the price and exchange-rate equations arbitrarily affect future values of the forcing variables, then it is impossible to recover the independent effect of structural disturbances on prices and exchange rates.

Table 2 is also consistent with univariate representations for the processes followed by p_c^*, q^*, k, and i^*. For these variables we specify univariate processes. We also use a univariate model for p_N^*, thereby treating the correlation between p_N^* and lags of m as an ex post statistical regularity that agents would not employ in forming forecasts of p_N^*. On the other hand, we specify agents' forecasts of future changes in m as depending also on U.S. interest-rate changes, as the date would seem to suggest. Table 3 contains the estimates, obtained by ordinary least squares, of our favored parsimonious representation (found by Box–Jenkins-type specification searches) of the stochastic process followed by the forcing variables.

The residuals of the maximum likelihood estimating Equations (13) and (14) consist only of current and predicted future values of both structural disturbances n and $\tilde{n}$. Again, we consider both the case in which these disturbances are white noise and the case in which they have first-order serial correlation.

If n and $\tilde{n}$ are white noise, their expected future values, conditional on currently available information, are 0. Under the normality assumption, maximum likelihood involves the minimization of the covariance matrix of the disturbances. These disturbances include those

Table 2. Significance Levels for the Absence of Causality From X to Y in Partially First-Differenced Vector Autoregressions

Variable X	Variable Y p_N^*	p_c^*	q^*	Δi^*	Δm	k
p_N^*	—	.73	.19	.74	.46	.58
p_c^*	.59	—	.22	.70	.15	.99
q^*	.47	.88	—	.65	.15	.59
Δi^*	.31	.92	.54	—	.001	.995
Δm	.004	.39	.49	.47	—	.76
k	.69	.76	.80	.99	.72	—
p_d	.30	.64	.64	.053	.36	.67
e	.98	.68	.65	.95	.47	.55

of the equations describing the forcing variables, as well as n and $\tilde{n}$.

The actual estimation is done, as described by Berndt, Hall, Hall, and Hausman (1974), by taking a single Newton step from the consistent estimates presented in Tables 1 and 3. To carry out this step, we numerically compute for each set of parameters the values of J_{21}, J_{22}, and ω's, and the ξ's to obtain the residuals of (13) and (14), taking into account the prediction formulas of Hansen and Sargent (1980). Maximum likelihood estimates and standard errors for the case in which n and $\tilde{n}$ are serially uncorrelated are presented in columns 1 and 2 of Table 4.

In the case in which n and $\tilde{n}$ have first-order serial correlation, maximum likelihood involves the minimization of the determinant of the covariance matrix that includes u_1 and u_2 [as defined in Eq. (15)] instead of n and $\tilde{n}$. Computation of u_1 and u_2 for the purpose of the minimization requires further transforming of the residuals of (13) and (14). The resulting estimates are reported in columns 3 and 4 of Table 4.

As can readily be seen, the estimates of Table 4 are very similar to those reported in Table 1. The estimates of the stochastic processes of the forcing variables, which are not reported, are virtually unchanged from Table 3.

The bottom two rows present likelihood ratio tests for the validity of the restrictions imposed by the model. The first, $2(L_1 - L_0)$, gives twice the difference of the log-likelihood of our constrained estimates (L_0) and those of a model that includes the same equations for the forcing variables but that leaves the price and exchange-rate equations unconstrained. It allows prices and exchange rates to be explained by all of the current and lagged values of the forcing variables that enter in our restricted model. Under the null hypothesis that the constraints derived from (13) and (14) are valid, this test statistic is distributed as a chi-square with degrees of freedom equal to the difference between the number of parameters in the two models. The second test, $2(L_2 - L_0)$, also considers the possibility that the forcing variables follow a different stochastic process. It thus contrasts the likelihood of our model with that of an unconstrained vector autoregression that includes six lags of all variables. All test statistics lead us to reject the constraints imposed by (13) and (14). Further evidence of misspecification is revealed by the DW statistics in columns 3 and 4, which indicate that the ex post u_1's have serial correlation equal to .7.

We now turn to the discussion of the parameter estimates. The values of ϕ_1 (ranging from 1.63 to 1.90 in Table 4) are significantly greater than 1, thus indicating that the effect of the exchange rate on the German price deflator is significantly different from 0. The coefficient is an increasing function of the income elasticity of money demand and of the terms of trade effect on aggregate demand.

The value and significance of ϕ_3 permits us to strongly

Table 3. Estimated Stochastic Process of Forcing Variables

Coefficients of	Variable X p^*_{ct}	p^*_{Nt}	k_t	q^*_t	$(m_t - m_{t-1})$	$(i^*_t - i^*_{t-1})$
X_{t-1}	1.247 (.093)	1.175 (.095)	.294 (.096)	.948 (.028)	.064 (.093)	.325 (.094)
X_{t-2}	−.283 (.093)	−.222 (.095)	.194 (.097)	—	−.086 (.097)	−.248 (.094)
X_{t-3}	—	—	.164 (.092)	—	.378 (.096)	—
$(i^*_{t-1} - i^*_{t-2})$	—	—	—	—	.068 (.113)	—
$(i^*_{t-2} - i^*_{t-3})$	—	—	—	—	−.238 (.118)	—
$(i^*_{t-3} - i^*_{t-4})$	—	—	—	—	.023 (.118)	—
$(i^*_{t-4} - i^*_{t-5})$	—	—	—	—	.268 (.113)	—
DW	2.03	2.08	2.06	1.85	2.00	2.00
R^2	.95	.93	.27	.91	.21	.12

NOTE: DW denotes Durbin–Watson statistics.

reject the version of the model with perfectly flexible prices. This is particularly interesting, since Hoffman and Schlagenhauf (1983) and Woo (1985) were unable to reject versions of the flexible-prices monetarist model that are very similar in all other respects to the one considered here. The increased ability of our test to reject the hypothesis of perfect price flexibility may be due to the use of a specific alternative hypothesis.

With $b > 0$, ϕ_4 is negative if $ad < 1$, where a is the income elasticity of money demand and d is the elasticity of aggregate demand to an increase in real balances. Thus $\phi_4 < 0$ is consistent with the U.S. evidence of Goldfeld (1973) and Rotemberg (1982b), who found values of a and d below 1.

The estimates of ϕ_6 are of the wrong sign and are insignificant in the version of the model with autocorrelated structural disturbances. The estimate of ϕ_7, which should be positive, is indeed positive in columns 3 and 4 of Table 4, but it is negative and insignificant in columns 1 and 2. Estimates α_1/ϕ_2 and α_2/ϕ_2 are unprecisely estimated in columns 1 and 3, thus validating the constraints we impose in columns 2 and 4.

Finally, Table 1 and Table 4 report the eigenvalues of the dynamic system consisting of Equations (8) and (12). These eigenvalues are computed from the parameter estimates. All estimates of Table 1 and the estimates of columns 2, 3, and 4 of Table 4 produce, as is required for our solution, two eigenvalues that are bigger than 1, while the other is smaller than 1.

4. SIMULATIONS

In this section, we use (13) and (14) to simulate the responses of the nominal exchange rate and the price level to a number of exogenous shocks.

One important empirical question that we can analyze with simulations of our model is whether the DM exchange rate overshoots its steady-state response to innovations in the German money stock. This type of question has received considerable attention in the theoretical and empirical literature since the seminal work of Dornbusch (1976), because these overreactions have the potential of explaining the volatility of exchange rates. Exchange-rate overshooting tends to be mitigated by two effects. The first is the immediate increase in the deflator for real-money balances due to the exchange-rate depreciation; the second is the increase in income and money demand associated with the worsening of the terms of trade. Note that, although the contemporaneous response of the price level reinforces the first effect, it decreases the importance of the second effect. By these two channels, the impact of the increase in money supply on the domestic interest rate is dampened, thus reducing the need for large exchange-rate adjustments (see Dornbusch 1976; Mussa 1982).

Figure 1 shows the responses to an innovation in the process followed by the German money stock. The simulation is carried out as follows. Starting at a steady state (in which all variables are normalized to 0), we produce a unit increase in the disturbance of the m_t autoregression at month 10. The price level and the exchange rate respond to such innovation according to our maximum likelihood estimates of Column 4 in Table 4. The money stock responds following the same maximum likelihood estimates. The figure shows that the money stock does increase further after the innovation and eventually reaches a permanently higher level.

Figure 1 also shows that the exchange rate rises more on impact than the price level, but it "undershoots" its new steady-state level. This undershooting results from the fact that money demand increases by more than money supply at an unchanged interest rate (so nominal interest rates must rise). Since the response of the de-

Table 4. Maximum Likelihood Estimates of the Price-Level and Exchange-Rate Euler Equations

	1	2	3	4
ϕ_1	1.637	1.737	1.903	1.806
	(.072)	(.061)	(.135)	(.116)
ϕ_3	.501	.500	.501	.501
	(.002)	(.002)	(.001)	(.001)
ϕ_4	−.108	−.093	−.129	−.271
	(.060)	(.060)	(.091)	(.095)
ϕ_5	.402	.441	.379	.416
	(.048)	(.044)	(.066)	(.059)
ϕ_6	−.004	−.003	−.007	−.001
	(.001)	(.001)	(.001)	(.002)
ϕ_7	−.001	−.0002	.004	.002
	(.001)	(.001)	(.002)	(.002)
α_1/ϕ_2	.011	—	.039	—
	(.009)		(.019)	
α_2/ϕ_2	−.000	—	.003	—
	(.001)		(.001)	
ϕ_2	—	89.788	—	39.378
		(19.862)		(16.240)
ρ_1	—	—	.119	.057
			(.077)	(.090)
ρ_2	—	—	.622	.649
			(.057)	(.052)
DW*				
n_t, u_{1t}	.880	.621	.860	.782
$\tilde{n}_t$, u_{2t}	.056	.627	1.789	1.896
Roots	1.630	1.746	1.892	1.798
	1.000 ± .036	1.012	1.091	1.061
		.986	.926	.950
$2(L_1 - L_0)$	502 [30 df]	314 [31 df]	250 [44 df]	214 [45 df]
$2(L_2 - L_0)$	1,277 [386 df]	1,090 [387 df]	902 [384 df]	866 [385 df]

NOTE: The sample is for June 1974–January 1983. Standard errors are in parentheses.
* Columns 1 and 2 contain the Durbin–Watson (DW) statistics for the estimates of n_t and $\tilde{n}_t$. Columns 3 and 4 contain the DW statistics for the estimates of u_{1t} and u_{2t}.

flator to an increase in money seems modest, we ascribe the relatively large response of money demand to a large effect of the terms of trade on output. This is consistent with the relatively large estimate of ϕ_1 that we obtain. Although we obtain undershooting, it is important to stress that the exchange rises much more than prices and the real exchange rate depreciates on impact (by .35% for a 1% increase in the money stock). Because of long-run neutrality, it gradually returns to its previous steady-state level.

Figure 2 shows the effects of a unit money innovation under the assumption that the money stock follows a random walk. This simulation is interesting because, as pointed out in the work of Currie (1984) and Meese and Singleton (1983), among others, the nature of the stochastic process followed by the money stock may have an important impact on the degree of overshooting. The figure shows that the lack of exchange-rate overshooting in Figure 1 is not due to the specific stochastic process followed by the money stock in Germany. Instead, it appears that the size of the coefficient ϕ_1 best explains the results of our simulations. A high value of ϕ_1 implies that exchange-rate depreciations, given the nominal interest rate, tend to reduce real-money balances relative to money demand. They do this by raising both output and the consumption deflator. As we argued previously, both of these factors prevent overshooting.

Figures 3 and 4 illustrate the effects of an increase in the U.S. treasury-bill rate. Given p_c^*, this amounts to a real-interest shock from the United States. As shown in Figure 3, the estimated reaction of m implies that an increase in U.S. interest rates is followed by a large monetary contraction in Germany. In the model, changes in i^* and m have contrasting effects on the real exchange rate. As discussed previously, falls in m tend to produce an immediate real appreciation, followed by a progressive depreciation. On the other hand, an increase in i^* tends to depreciate the real exchange rate on impact. The latter effect dominates. In addition, the sharp monetary contraction generates a price deflation in the long run. As in the case of an innovation in m, the estimated process for i^* shows that interest-rate shocks have a permanent component. A permanent increase in the foreign real interest rate brings about a permanent real exchange-rate depreciation, because at full employment higher interest rates are associated with

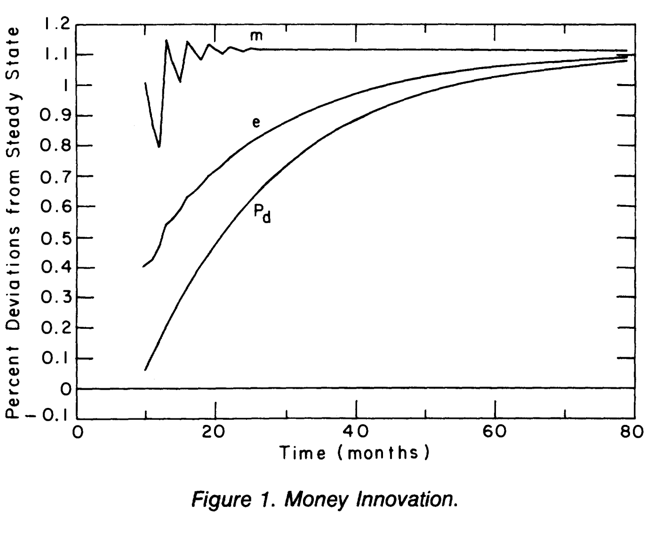

Figure 1. Money Innovation.

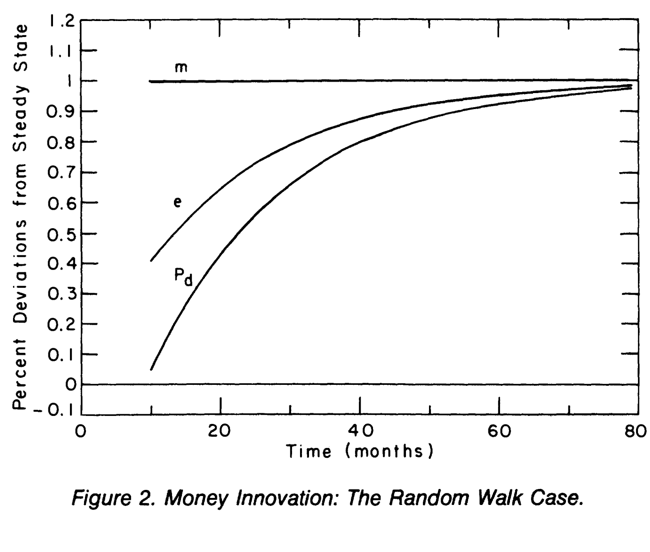

Figure 2. Money Innovation: The Random Walk Case.

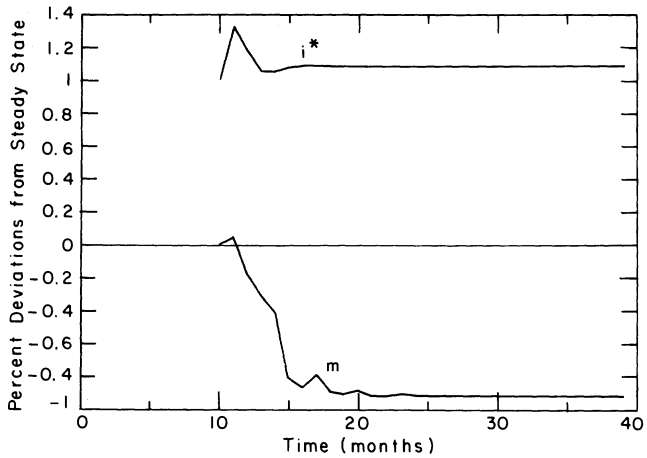

Figure 3. U.S. Treasury-Bill Rate Innovation; U.S. Treasury-Bill Rate and German Money Stock.

lower real balances and, therefore, lower domestic demand. The real exchange-rate depreciation crowds in net foreign demand for domestic output.

Figures 3 and 4 also suggest that the sizable real depreciation of the DM/dollar rate observed from 1980 to 1985 cannot be ascribed to higher U.S. real interest rates alone; even the steady-state response of the real exchange rate to a 1% real interest rate shock is only 1.92%. This suggests that other variables, like monetary shocks in Germany and various supply shocks, might also have played a role in the recent rate experience. To illustrate the ability of all of the variables in our model to track the recent real depreciation of the DM/dollar rate, we compare predictions (for our detrended data) generated by the estimates of column 4 in Table 4 to the actual experience from June 1980 to July 1985. Note that our parameters are estimated using data only through January 1983.

Figure 5 plots actual and simulated values of $p_c^* + e - p_d$. The figure shows that the model traces out many of the turning points of the DM/dollar real exchange rate from 1980 to 1985. It is unable to reproduce the trend depreciation, however; only half of the 30% cumulated real depreciation of the DM from June 1980 to July 1985 is reproduced by the model. In addition, the sizable swing from June 1984 to July 1985 is also significantly underestimated.

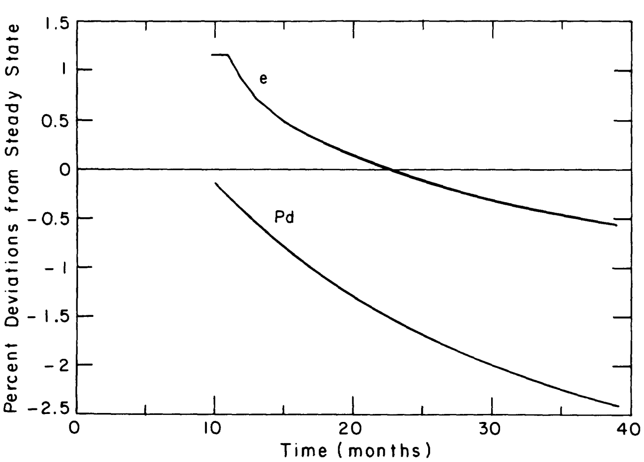

Figure 4. U.S. Treasury-Bill Rate Innovation; German Price Level and DM Exchange Rate.

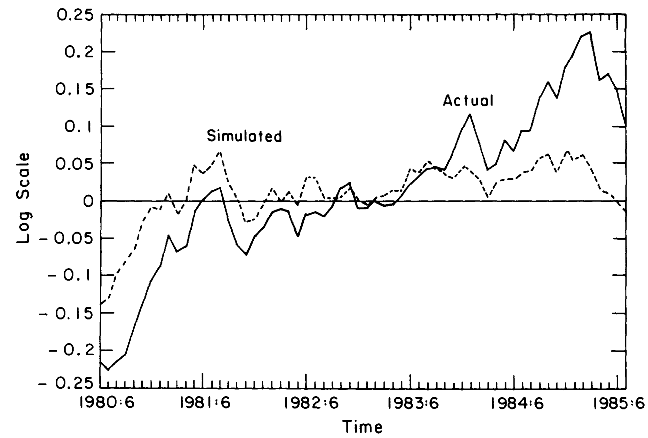

Figure 5. The DM Real Exchange Rate (simulation of the 1980–1985 episode).

5. CONCLUDING REMARKS

This article specifies and estimates a rational-expectations model of sticky prices in an open economy. The results are mixed. On the one hand, the model is not rejected when estimated by instrumental variables, and it produces reasonable parameter estimates. On the other hand, the cross-equation restrictions that require that agents, when they are forecasting the future values of various forcing variables, use the estimated stochastic processes for these variables are rejected. These apparently inconsistent results might be due to the different power of the two tests, to the failure of the structural disturbances to follow an unconditional normal distribution, and to errors in the specification of either the structural equations or the forecasting equation for the forcing variables. Thus there is, unfortunately, little that we can conclude from this negative test result.

Since at least in some respects our model performs well, we believe that empirical research on well-specified structural exchange-rate models is worth pursuing further. Thus we do not feel that this style of research is necessarily doomed by the findings of Meese and Rogoff (1983) that simple structural exchange-rate models are worse at predicting exchange rates than random walks. For instance, although our model does not predict the level of the real exchange rate particularly well, it appears able to explain several of its turning points. This suggests that simple comparisons of root mean-squared prediction errors may be an incomplete method for gauging a model's usefulness. It would seem that more research on methods for validating the out-of-sample forecasting ability of different models, as well as research on improvements in structural models, is needed before one can decide whether the ideas of current exchange-rate theories are useful for understanding the behavior of actual exchange rates.

Improvements along these lines could include endogenizing some of the forcing variables, like the real-wage rate and, possibly, the money supply. Moreover, the explicit inclusion of real interest rate in aggregate demand might improve the fit of the model. These extensions, however, considerably increase the size and complexity of the model and would make maximum likelihood estimation quite expensive.

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Robert Cumby and John Huizinga kindly provided us with software for generalized instrumental variables estimation. Financial support from the John M. Olin Foundation, the Fondazione Einandi, and the National Science Foundation is gratefully acknowledged.

APPENDIX: THE DATA

This article presents an empirical analysis of the dollar/DM exchange rate, but, unlike earlier work, it aggregates the rest of the world (for Germany) as a single dollar area.

Data for P_c^* and Q^* are computed using geometrically weighted averages of the individual series from the following countries: the United States (weight .157), France (weight .275), Italy (weight .184), the Netherlands (weight .252), and the United Kingdom (weight .132). The weights are the average from 1974 to 1981 of the ratio of the value of imports plus exports of each country with Germany over the total trade of Germany with these countries. In 1981, the five countries just listed represented 45% of the total trade of Germany. Individual countries' indexes for P_c and q are aggregated using the respective exchange rates vis-à-vis the U.S. dollar.

Most of the data is obtained from the *International Financial Statistics* (IFS) tape. The index of wages in Germany and the index of intermediate inputs prices are computed using data from the Deutsche Bundesbank's *Monthly Report* and various statistical supplements.

The exchange rate is the price of a U.S. dollar in DM's, line rf in IFS (average over the month).

The domestic price level is line 63 of IFS, the index of wholesale prices.

Variable P_c^* is computed aggregating the various countries' indexes of consumer prices, line 64 in IFS.

Variable P_N^* is a weighted average of the IFS index of all commodities prices excluding oil, line 76a, and the index of the dollar price of oil for Saudi Arabia, line 76aa.

The weights are chosen from the geographical composition of Germany's imports, using data from the Deutsche Bundesbank's *Monthly Report.* For the period 1974–1982, we computed the average share of imports from the Organization of Petroleum Exporting Countries (OPEC) in total imports less finished goods. The average value share of imports from OPEC is 22%.

Variable i^* is the U.S. treasury-bill rate, line 60c in IFS.

Variable Q^* is the weighted average of industrial production indexes, line 66c in IFS, expressed in dollar terms, by dividing each country's index by the real dollar exchange rate.

Variable M is line 34 in IFS.

Variable K is the index of wage costs in industry, per manhour, divided by the CPI. The former is from the Bundesbank's *Monthly Bulletin;* the latter is IFS, line 64.

Theory requires that all series be realizations of stationary stochastic processes. The mean and trend of all series have been removed by regressing the log of each against a constant, time and time squared. For the seasonally unadjusted data, 11 monthly dummies were also included in the regression.

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