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# A Monetary Equilibrium Model with Transactions Costs

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This paper presents the competitive equilibrium of an economy in which people hold money for transactions purposes. It studies both the steady states that result from different rates of monetary expansion and the effects of such non-steady-state events as an open-market operation. Even though the model features no uncertainty and perfect foresight, open-market operations affect aggregate output. In particular, a simultaneous increase in money and governmental holdings of capital temporarily raises aggregate capital and output while it lowers the real rate of interest on capital.

## I. Introduction

The objective of this paper is to study the competitive equilibrium of an economy in which people hold money for transactions purposes. As in the models of Baumol (1952), Tobin (1956), Stockman (1981), Jovanovic (1982), and Townsend (1982), but in contrast to those of Grandmont and Younes (1973), Lucas (1980), and Helpman (1981), households are allowed to hold interest-bearing capital in addition to barren money. The main advantage of the present model is that it is

I wish to thank Stanley Fischer, Timothy Kehoe, Robert Pindyck, Paul Romer, Robert Townsend, and especially Sanford Grossman for helpful comments and suggestions while retaining full responsibility for any remaining errors. I am also grateful to the National Science Foundation for its support through grant no. SES-8209266. This paper and those written independently by Grossman and Weiss (1982) and Grossman (1982) contain a very similar argument. However, the three papers differ in their details and should be viewed as complementary. Some of these differences are sketched in footnotes 2 and 6.

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able to shed light on the effects of such non-steady-state events as open-market operations.

Households pick the path of consumption optimally from their point of view. Because it is costly to carry out financial transactions, people visit their financial intermediary only occasionally. However, I do not let households pick optimally the length of the period during which they do not visit their bank. For tractability, the assumption is made that households have a constant interval during which they carry out no financial transactions.

A crucial feature of this paper is that, as in all free-market economies, different households visit their banks at different times. This leads to conclusions that are strikingly "Keynesian." Government interventions, and in particular open-market operations, have the ability to affect aggregate output and the real rate of interest. This is true even though the model features no uncertainty, full information, perfect foresight, and perfectly competitive markets for goods and money. The central finding of this paper is that a one-period monetary expansion leads to a lower real interest rate and a higher level of output for some time.

The paper proceeds as follows: Section II presents the model. It shows the maximization problems of the households and the firms, as well as the institutional environment. Section III presents the perfect-foresight equilibrium of the economy. It is a difference equation that, under certain conditions, exhibits saddle-path stability near the steady state that has positive consumption. Section IV studies steady-state inflation and welfare. It establishes that the level of output is independent of the rate of inflation but that inflation affects welfare negatively by distorting the intertemporal consumption decisions. Section V is the heart of the paper since it discusses monetary policy outside the steady state. Finally, Section VI presents some conclusions.

## II. The Model

There is only one good that can be both consumed and invested. Total output produced by competitive firms in period  $t$  ( $Q_t$ ) depends, via a constant-returns-to-scale production function, on the amount of labor hired at  $t$  and on  $K_{t-1}$ , the amount of the good that was produced but not consumed at  $t - 1$ . Since an amount of labor  $\bar{L}$  is assumed to be supplied inelastically,

$$Q_t = \bar{L}f\left(\frac{K_{t-1}}{\bar{L}}\right), \quad (1)$$

where  $f$  is an increasing and concave function. For notational simplicity  $Q_t$  is total output and includes the depreciated value of  $K_{t-1}$ . Workers are assumed to be paid their marginal product. Therefore, the total return, denominated in period  $t$  goods, from forgoing the consumption of an additional good at  $t - 1$  is given by  $1 + r_{t-1}$ , where<sup>1</sup>

$$1 + r_{t-1} = f' \left( \frac{K_{t-1}}{\bar{L}} \right). \quad (2)$$

There are  $2n$  households. At time  $t$ , the households are assumed to maximize the utility function given by

$$V_t = \sum_{\tau=t}^{\infty} \rho^{\tau-t} \ln C_{\tau}^i, \quad (3)$$

where  $C_{\tau}^i$  is the consumption of household  $i$  at time  $\tau$  and  $\rho$  is a discount factor. The households have access to two assets, money and claims on capital. Money is the only asset with which goods can be bought. Moreover, visits to the financial intermediary for the purpose of converting claims on capital into money are costly. Therefore, as in the inventory theoretical models of Baumol (1952) and Tobin (1956), households engage in these visits only sporadically. In this paper it will be assumed at the outset that households exchange capital for money every two periods. The assumption that households do not change the timing of their financial transactions in response to events is made mainly for tractability. Except in stationary environments, it is very difficult to solve for the optimal timing of these visits, particularly when households pick their consumption path optimally.

Without loss of generality, suppose that household  $i$  engages in financial transactions in the "even" periods,  $t, t + 2, t + 4, \dots$ . At these dates it withdraws an amount  $M_{\tau}^i$  of money balances that must be sufficient to pay for its consumption at  $\tau$  and  $\tau + 1$ :

$$M_{\tau}^i = P_{\tau} C_{\tau}^i + P_{\tau+1} C_{\tau+1}^i, \quad (4)$$

where  $P_{\tau}$  is the nominal price of the consumption and investment good at  $\tau$ . Note that  $M_{\tau}^i$  units of money are withdrawn at the beginning of period  $\tau$  and are thus available to make purchases at  $\tau$ .<sup>2</sup>

<sup>1</sup> Primes denote first derivatives, while double primes denote second derivatives.

<sup>2</sup> Grossman (1982) and Grossman and Weiss (1982) assume instead that money is withdrawn at the end of the period. Thus it is only available for consumption the following period.

The lifetime budget constraint of the household at  $t$  is given in terms of the household's monetary withdrawals:

$$\begin{aligned} \sum_{\tau=0}^{\infty} \frac{M_{t+2\tau}^i}{P_{t+2\tau}} / \prod_{i=0}^{2\tau-1} (1 + r_{t+i}) \\ = \left[ \sum_{\tau=0}^{\infty} Y_{t+\tau}^i / \prod_{i=0}^{\tau-1} (1 + r_{t+i}) \right] + K_{t-1}^i (1 + r_{t-1}) \quad (5) \\ - \left[ \sum_{\tau=0}^{\infty} B / \prod_{i=0}^{2\tau-1} (1 + r_{t+i}) \right], \end{aligned}$$

where  $K_t^i$  are the claims on capital of household  $i$  at  $t$ ,  $B$  is the real cost of visiting the financial intermediary, and  $Y_\tau^i$  is the noninvestment income of the household at  $\tau$ . This noninvestment income includes labor income as well as taxes and transfers from the government. Note that (5) explicitly assumes that noncapital income is directly invested in claims on capital. This assumption considerably simplifies the analysis.

The optimal path of consumption is found in two steps. First, I derive consumption at  $\tau$  and  $\tau + 1$  as a function of  $M_\tau^i$ . Then I derive the optimal values of the sequence of monetary withdrawals.

The first step requires the maximization of

$$\ln C_\tau^i + \rho \ln C_{\tau+1}^i \quad (6)$$

subject to (4). This yields<sup>3</sup>

$$C_{\tau+1}^i = \frac{\rho P_\tau}{P_{\tau+1}} C_\tau^i = \frac{\rho}{1 + \rho} \frac{M_\tau^i}{P_{\tau+1}}. \quad (7)$$

Using (7), the expression in (6) is given by

$$\begin{aligned} \ln C_\tau^i + \rho \ln C_{\tau+1}^i = (1 + \rho) \ln \left( \frac{M_\tau^i}{P_\tau} \right) + \rho \ln \rho \\ - (1 + \rho) \ln (1 + \rho) - \rho \ln \left( \frac{P_{t+1}}{P_t} \right). \quad (8) \end{aligned}$$

Equation (8) asserts that the appropriately weighted sum of the instantaneous utilities at  $\tau$  and  $\tau + 1$  increases with  $M_\tau^i/P_\tau$  but is negatively affected by inflation between  $\tau$  and  $\tau + 1$ .

<sup>3</sup> The analysis assumes that the holdings of money by household  $i$  from  $\tau$  to  $\tau + 1$  are positive. Here this is guaranteed by the fact that nonpositive holdings of money would induce nonpositive consumption and hence utility equal to minus infinity. However, if labor income were paid in the form of money, the constraint that monetary holdings be nonnegative might become binding.

Substituting (8) into (3) and using (5), one obtains

$$V_t = \sum_{k=0}^{\infty} \rho^{2k} \left[ (1 + \rho) \ln \frac{M_{t+k}^i}{P_{t+k}} + \rho \ln \rho - (1 + \rho) \ln (1 + \rho) - \rho \ln \frac{P_{t+k+1}}{P_{t+k}} \right]. \quad (9)$$

This expression must now be maximized with respect to the sequence of monetary withdrawals subject to the lifetime budget constraint (5). This maximization yields

$$- \frac{1}{M_{t+2k}^i/P_{t+2k}} + \frac{\rho^2(1 + r_{t+2k})(1 + r_{t+2k+1})}{M_{t+2k+2}^i/P_{t+2k+2}} = 0, \quad (10)$$

as well as the transversality condition found, for instance, in Arrow and Kurz (1970):

$$\lim_{T \rightarrow \infty} \frac{\rho^T K_T^i}{M_T^i/P_T} = 0. \quad (11)$$

Using (7), (10) becomes

$$\frac{C_{t+2k+2}^i}{C_{t+2k}^i} = \rho^2(1 + r_{t+2k})(1 + r_{t+2k+1}). \quad (12)$$

Note that both (12) and (7) state that the marginal rate of substitution times a rate of return is equal to one. The important distinction between the two is that in (7) the rate of return is the rate of return on money, while in (12) it is the rate of return on capital. Stochastic versions of (12) have been statistically rejected using aggregate U.S. data by Mankiw (1981) and Hansen and Singleton (1982). Their rejections may be due in part to their neglect of the fact that in the presence of the transactions motive for holding money, the ratio of marginal utilities of consumption separated by different time intervals is related to rates of return of assets with different characteristics.

The government in this model has no expenditures. However, it levies taxes, issues money, and holds capital. The evolution of the capital held by the government is given by<sup>4</sup>

$$K_{\tau+1}^G = f'(K_{\tau})K_{\tau}^G + \frac{M_{\tau+1} - M_{\tau}}{P_{\tau+1}} + T_{\tau+1}, \quad (13)$$

where  $T_{\tau+1}$  is the real taxes levied at  $\tau + 1$ ,  $K_{\tau}^G$  is the government's real holdings of capital at  $\tau$ , and  $M_{\tau}$  is high-powered money at  $\tau$ . An increase in  $M_{\tau+1}$  relative to  $M_{\tau}$  will be called an open-market purchase

<sup>4</sup> For simplicity, I ignore the transaction costs incurred by the government when it engages in an open-market operation.

and therefore the domain of monetary policy. Instead, a simultaneous change of  $T_{\tau+1}$  and  $K_{\tau+1}^G$  will be considered a type of fiscal policy. The government also requires that between periods  $\tau$  and  $\tau + 1$  (overnight) the monetary assets and liabilities of the financial intermediaries be equal to total high-powered money,  $M_\tau$ . This is a form of a 100 percent reserve requirement in which the only form taken by high-powered money is deposits of intermediaries at the central bank. Hence, individuals' money consists of claims on the intermediaries only.

Financial intermediaries receive the household's income and invest it in claims on capital. They are also allowed to issue a certain quantity of money. The intermediaries are compensated for their services with the brokerage fees,  $B$ , of (5). Their function can best be understood by following their transactions in detail.

Between periods  $\tau$  and  $\tau + 1$  the financial intermediaries have as their assets the household's claims on capital as well as  $M_\tau$  units of money. Their liabilities are the household's claims on capital and the monetary assets of the households. The households that visit the intermediaries at  $\tau + 1$  do not carry over any money between  $\tau$  and  $\tau + 1$  if the nominal rate of interest is positive, which is assumed throughout. So the monetary liabilities of the intermediaries between  $\tau$  and  $\tau + 1$  consist solely of the amount withdrawn but not spent by the households that visit the intermediaries at  $\tau$ . These liabilities cannot, by law, exceed  $M_\tau$ . The intermediaries would, of course, like to issue more money as long as the nominal rate of interest is positive.

At the beginning of  $\tau + 1$ , the financial intermediaries issue whatever amount of money ( $nM_{\tau+1}^i$ , where  $n$  is the number of households) is required by the households that visit them at  $\tau + 1$ . Meanwhile, their nonmonetary assets and liabilities have grown since the firms now must pay interest on their debt as well as pay the households their labor income. Both these payments are made through the intermediaries. Therefore, the intermediaries' consolidated balance sheet measured in nominal dollars is given by table 1, where  $K_\tau^P$  and  $Y_\tau^L$  are the amount of capital owned by the private sector and labor income at  $\tau$ , respectively.

TABLE 1

CONSOLIDATED BALANCE SHEET OF THE INTERMEDIARIES AT THE BEGINNING OF  $\tau + 1$

	Assets	Liabilities
Nonmonetary	$\left[ K_\tau^P f' \left( \frac{K_\tau}{L} \right) + Y_{\tau+1}^L \right] P_{\tau+1}$	$\left[ K_\tau^P f' \left( \frac{K_\tau}{L} \right) + Y_{\tau+1}^L \right] P_{\tau+1} - nM_{\tau+1}^i$
Monetary	$M_\tau$	$nM_{\tau+1}^i + M_\tau$

During the period  $\tau + 1$ , the households make purchases whose total value is  $P_{\tau+1}C_{\tau+1}$ , where  $C_{\tau+1}$  is total consumption at  $\tau + 1$ . These purchases are paid with money that the firms return to the intermediaries at the end of the period in partial payment of their obligations. The firms also sell, for money that they likewise return to the intermediaries,  $(M_{\tau+1} - M_{\tau})/P_{\tau+1}$  units of capital to the government. Finally, the government levies taxes directly on the nonmonetary assets held by the households at the intermediaries. Therefore the consolidated balance sheet of the intermediaries at the end of  $\tau + 1$  is given by table 2.

The government requires that the monetary liabilities be equal to  $M_{\tau+1}$ . Thus,  $nM_{\tau+1}^i = P_{\tau+1}C_{\tau+1} + M_{\tau+1} - M_{\tau}$ . Substituting for  $nM_{\tau+1}^i$  in the nonmonetary liabilities, it is clear that the nonmonetary liabilities equal the nonmonetary assets. Moreover,

$$\begin{aligned} P_{\tau+1} \left[ K_{\tau}^P f' \left( \frac{K_{\tau}}{L} \right) + Y_{\tau+1}^L - C_{\tau+1} - T_{\tau+1} \right] - (M_{\tau+1} - M_{\tau}) \\ = P_{\tau+1} \left[ (K_{\tau}^P + K_{\tau}^G) f' \left( \frac{K_{\tau}}{L} \right) + Y_{\tau+1}^L - K_{\tau+1}^G - C_{\tau+1} \right] \quad (14) \\ = P_{\tau+1} (Q_{\tau+1} - C_{\tau+1} - K_{\tau+1}^G) = P_{\tau+1} K_{\tau+1}^P, \end{aligned}$$

where the first equality is obtained using (13). The second equality follows from the fact that  $K_{\tau}^P + K_{\tau}^G = K_{\tau}$  and that, under constant returns to scale, the value of output is equal to factor payments. Finally, the last equality results from using the identity that makes output,  $Q_{\tau}$ , equal to consumption,  $C_{\tau}$ , plus investment,  $K_{\tau}$ . Hence, again, the nonmonetary assets and liabilities of the intermediaries between  $\tau + 1$  and  $\tau + 2$  are equal to the private holdings of capital.

I also assume that half the households ( $n$ ) visit the intermediaries in the even periods  $t, t + 2, t + 4, \dots$ , and the other half carry out their financial transactions in the odd periods. The fact that only a subset of the households visit financial intermediaries in any given day is one of the main features of reality that this paper seeks to reflect. It also is a feature of the steady states studied by Jovanovic (1982). It turns out that the assumption that households stagger their financial transactions is crucial to ensure that open-market operations have real effects.

### III. Equilibrium

The equilibrium for this economy is a path for the price level and for the real rate of interest such that households maximize utility and firms maximize profits using these prices and the following two conditions are met.

TABLE 2  
CONSOLIDATED BALANCE SHEET OF THE INTERMEDIARIES AT THE END OF  $\tau + 1$

	Assets	Liabilities
Nonmonetary	$P_{\tau+1} \left[ K_{\tau}^L f' \left( \frac{K_{\tau}}{L} \right) + Y_{\tau+1}^L - C_{\tau+1} - T_{\tau+1} \right] - (M_{\tau+1} - M_{\tau})$	$P_{\tau+1} \left[ K_{\tau}^L f' \left( \frac{K_{\tau}}{L} \right) + Y_{\tau+1}^L - T_{\tau+1} \right] - nM_{\tau+1}^i$
Monetary	$M_{\tau} + (M_{\tau+1} - M_{\tau})$	$nM_{\tau+1}^i + M_{\tau} - P_{\tau+1} C_{\tau+1}$

1. The sum of consumption and capital demanded at  $\tau$  by the households and capital demanded by the government at  $\tau$  is equal to output at  $\tau$ :

$$C_\tau + K_\tau^P + K_\tau^G = \bar{L}f\left(\frac{K_{\tau-1}}{\bar{L}}\right). \quad (15)$$

2. The amount of money that households that visit the intermediaries at  $\tau$  want to hold between  $\tau$  and  $\tau + 1$  must be equal to  $M_\tau$ . Hence, the total expenditures at  $\tau + 1$  by households that visit the financial intermediaries at  $\tau$  must be equal to  $M_\tau$ .

Let  $C_\tau^\tau$  and  $C_\tau^{\tau-1}$  be the consumptions at  $\tau$  of households that visit the financial intermediary at  $\tau$  and  $\tau - 1$ , respectively. Then condition (2) requires that

$$nP_\tau C_\tau^{\tau-1} = M_{\tau-1}. \quad (16)$$

Therefore, using (7),

$$n\rho P_\tau C_\tau^\tau = M_\tau. \quad (17)$$

Therefore,

$$C_\tau^{\tau-1} = \frac{\rho M_{\tau-1}}{M_\tau} C_\tau^\tau, \quad (18)$$

and aggregate consumption at  $\tau$ ,  $C_\tau$ , is given by

$$C_\tau = n\left(1 + \frac{\rho M_{\tau-1}}{M_\tau}\right) C_\tau^\tau. \quad (19)$$

Using (19), (11), and the equilibrium condition (15), one obtains the difference equation that governs the evolution of aggregate capital:

$$\begin{aligned} \bar{L}f\left(\frac{K_{\tau+2}}{\bar{L}}\right) - K_{\tau+3} &= \rho^2 \frac{1 + \rho(M_{\tau+2}/M_{\tau+3})}{1 + \rho(M_\tau/M_{\tau+1})} f'\left(\frac{K_{\tau+1}}{\bar{L}}\right) f'\left(\frac{K_{\tau+2}}{\bar{L}}\right) \\ &\times \left[\bar{L}f\left(\frac{K_\tau}{\bar{L}}\right) - K_{\tau+1}\right], \end{aligned} \quad (20)$$

$$\tau = t - 1, t, t + 1, \dots$$

Knowledge of the sequence of capitals provides the sequence of rates of return by (2), the aggregate consumptions by (15), the sequence of individual consumptions by (19), and the sequence of prices by (17). The equilibrium is thus a third-order nonlinear difference equation with, so far, only one initial condition, namely,  $K_{t-1}$ . It will be shown below that equation (20), which is a necessary condition for an equilibrium, establishes that open-market operations are nonneutral. Before studying this question, however, I focus on the existence and

uniqueness of an equilibrium that converges to a steady state with positive consumption. As long as  $M_\tau/M_{\tau+1}$  converges to a constant, the steady-state values of capital ( $K^*$ ) have the following property:

$$\left[ \bar{L}f\left(\frac{K^*}{\bar{L}}\right) - K^* \right] \left\{ 1 - \rho^2 \left[ f'\left(\frac{K^*}{\bar{L}}\right) \right]^2 \right\} = 0.$$

The steady-state values of  $K$  do not depend on the rate of monetary growth. There are two types of steady states. Those with zero consumption are such that output  $\bar{L}f(K^*/\bar{L})$  is equal to investment,  $K^*$ . These are ruled out by the transversality condition (11). The only steady state with positive consumption has the property that  $\rho f'(\bar{K}) = 1$ ; the product of the discount rate times the marginal product of capital is unity. Unfortunately, I cannot establish the existence or uniqueness of paths that converge to  $\bar{K}$ . I can only present a local result, namely, that the linearized version around  $\bar{K}$  of (20) is such that, under certain conditions, a unique path that converges to  $\bar{K}$  exists. The linearized version of (20) around  $\bar{K}$ , assuming money growth is constant, is

$$\begin{aligned} & (K_{\tau+3} - \bar{K}) - \left\{ f'(\bar{K}) - \rho \left[ f\left(\frac{\bar{K}}{\bar{L}}\right) - \frac{\bar{K}}{\bar{L}} \right] f''\left(\frac{\bar{K}}{\bar{L}}\right) \right\} (K_{\tau+2} - \bar{K}) \\ & - \left\{ 1 - \rho \left[ f\left(\frac{\bar{K}}{\bar{L}}\right) - \frac{\bar{K}}{\bar{L}} \right] f''\left(\frac{\bar{K}}{\bar{L}}\right) \right\} (K_{\tau+1} - \bar{K}) + f'\left(\frac{\bar{K}}{\bar{L}}\right) (K_\tau - \bar{K}) = 0 \end{aligned} \quad (21)$$

or  $(1 - \lambda_1 L)(1 - \lambda_2 L)(1 - \lambda_3 L)(K_\tau - \bar{K}) = 0$ , where  $L$  is the lag operator and the roots  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  have the following properties:

$$\begin{aligned} \lambda_1 \lambda_2 \lambda_3 &= -f'\left(\frac{\bar{K}}{\bar{L}}\right), \\ \lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_1 \lambda_3 &= -1 + \rho \left[ f\left(\frac{\bar{K}}{\bar{L}}\right) - \frac{\bar{K}}{\bar{L}} \right] f''\left(\frac{\bar{K}}{\bar{L}}\right), \\ \lambda_1 + \lambda_2 + \lambda_3 &= f'\left(\frac{\bar{K}}{\bar{L}}\right) - \rho \left[ f\left(\frac{\bar{K}}{\bar{L}}\right) - \frac{\bar{K}}{\bar{L}} \right] f''\left(\frac{\bar{K}}{\bar{L}}\right). \end{aligned}$$

Inspection of these equations reveals that one of the roots, say  $\lambda_1$ , is equal to minus one,<sup>5</sup> while the other two roots are such that  $(\lambda_2 - 1)(\lambda_3 - 1)$  is negative,  $\lambda_2 \lambda_3$  is positive, and  $(\lambda_2 + \lambda_3)$  is positive. Therefore,  $\lambda_2$  and  $\lambda_3$  are both positive and lie on opposite sides of the unit circle. There is one stable root, say  $\lambda_3$ . This root corresponds to the

<sup>5</sup> To see that one of the roots is equal to minus one, it suffices to check that in (21) one (the coefficient of  $K_{\tau+3}$ ) minus the coefficient of  $K_{\tau+2}$  plus the coefficient of  $K_{\tau+1}$  minus the coefficient of  $K_\tau$  is equal to zero.

initial condition that gives, at  $t$ , the historical level of  $K_{t-1}$ . The root bigger than one,  $\lambda_2$ , is such that, if the initial conditions are not chosen correctly,  $K$  either explodes or implodes, contradicting the transversality condition (11). The third root,  $\lambda_1$ , must ensure that the path is feasible not only for the economy as a whole but also for the two types of households. This initial condition can be obtained as follows. Using (10) and (7) for a household that withdraws money at  $t$ , (5) becomes

$$\begin{aligned} \frac{(1 + \rho)}{1 - \rho^2} C_t^t = & \left[ \sum_{\tau=0}^{\infty} Y_{t+\tau}^t / \prod_{i=0}^{\tau-1} (1 + r_{t+i}) \right] + K_{t-1}^t (1 + r_{t-1}) \\ & - \left[ \sum_{\tau=0}^{\infty} B / \prod_{i=0}^{2\tau-1} (1 + r_{t+i}) \right], \end{aligned} \quad (22)$$

where  $Y_{t+\tau}^t$  and  $K_{t-1}^t$  are the noncapital income earned at  $t + \tau$  and the capital holdings at  $t - 1$  of an agent who visits the intermediaries at  $t$ . The sequence of  $Y$ 's in (22) is endogenous, so (22) is not an appropriate initial condition. Instead, consider the difference between  $C_{t+1}^{t+1}$  and  $C_t^t(1 + r_t)$ , where  $C_t^t$  is given by (22) and  $C_{t+1}^{t+1}$  is given by the corresponding equation for the individual who goes to the intermediaries at  $t + 1$ :

$$\begin{aligned} [C_{t+1}^{t+1} - C_t^t(1 + r_t)] \frac{(1 + \rho)}{1 - \rho^2} = & (Y_{t+1}^{t+1} - Y_t^t)(1 + r_t) \\ & + \left[ \sum_{\tau=0}^{\infty} (Y_{t+1+\tau}^{t+1} - Y_{t+1+\tau}^t) / \prod_{i=0}^{\tau-1} (1 + r_{t+i+1}) \right] \\ & + (K_{t-1}^{t+1} - K_{t-1}^t)(1 + r_{t-1})(1 + r_t) + B(1 + r_t) \\ & + \left[ \sum_{\tau=1}^{\infty} B / \prod_{i=1}^{2\tau-1} (1 + r_{t+i}) \right] - \left[ \sum_{\tau=0}^{\infty} B / \prod_{i=0}^{2\tau-1} (1 + r_{t+i+1}) \right]. \end{aligned}$$

Assuming  $Y_{t+1}^{t+1}$  is equal to  $Y_{t+1}^t$ , ignoring the transaction costs  $B$ , and using (19) this equation becomes

$$\begin{aligned} \frac{1 + \rho}{n} \left\{ \frac{\bar{L}f(K_t/\bar{L}) - K_{t+1}}{1 + \rho(M_t/M_{t+1})} - \frac{f'(K_t/\bar{L})[\bar{L}f(K_{t-1}/\bar{L}) - K_t]}{1 + \rho(M_{t-1}/M_t)} \right\} \\ = (K_{t-1}^{t+1} - K_{t-1}^t) f' \left( \frac{K_{t-1}}{\bar{L}} \right) f' \left( \frac{K_t}{\bar{L}} \right) (1 - \rho^2), \end{aligned} \quad (23)$$

which is another initial condition.

Unfortunately, as pointed out by Samuelson (1947), for instance, the presence of a root with modulus one in the linearized equation is consistent with both stability and instability of the nonlinear equation.

The higher-order terms matter. The oscillatory root  $\lambda_1$  could “correspond” to convergent, oscillatory, or divergent behavior. In other words, the stable manifold of (20) can be either two- or one-dimensional. In the former case, the initial condition given by (23) ensures that there is a unique equilibrium that converges to  $\bar{K}$ .<sup>6</sup> In the latter case, there may exist no convergent equilibrium that satisfies (23).<sup>7</sup> In other words, the path that converges to  $\bar{K}$  may not satisfy (23). However, this convergent path will still be an equilibrium if the government carries out lump-sum transfers that ensure that, eventually, the wealth of the two types of agents is the same so that consumption is constant.

#### IV. Inflation, Steady States, and Welfare

In Section III, I established that there is no Tobin effect in this model. Independently of the value of  $M_{t+1}/M_t$ , the unique steady-state value of the capital stock that involves positive consumption is  $\bar{K}$ . This does not mean that inflation has no real effects. In particular, the steady-state rate of growth of the money stock affects the path of individual consumption, the level of welfare, and the income velocity of money.

Before studying the effects of inflation, however, it must be ascertained that an inflationary path is consistent with the government’s budget constraint (14). I will assume that the government lets the money stock grow at the rate  $m$  so that  $M_\tau/M_{\tau-1} = 1 + m$ . With the new money, the government buys capital, which it redistributes in lump-sum fashion. These lump-sum redistributions affect none of the conditions used to derive (20). So the amount of capital held by the government can be arbitrarily set to some constant.

I now compute the rate of inflation, which corresponds to a given rate of growth of money. Equation (17) establishes that the steady-state rate of inflation,  $\pi$ , is given by

$$\frac{P_{\tau+1}}{P_\tau} = 1 + \pi = \rho \frac{C_\tau^\tau}{C_{\tau-1}^{\tau-1}}, \tag{24}$$

where in the steady state neither  $C_\tau^\tau$  nor  $C_{\tau-1}^{\tau-1}$  depends on  $\tau$ . Therefore, using (18),

$$1 + \pi = \frac{M_\tau}{M_{\tau-1}} = 1 + m. \tag{25}$$

<sup>6</sup> Grossman (1982) and Grossman and Weiss (1982) consider a situation with no capital and in which output is constant. In their model, the oscillatory root is always convergent.

<sup>7</sup> The properties of equilibria in this case are unknown. However, it is possible that the equilibrium settles down to an oscillation between two levels of capital  $\hat{K}$  and  $\bar{K}$  such that  $\rho^2 f'(\hat{K})f'(\bar{K})$  is equal to one. Such oscillations satisfy (20) as long as consumption oscillates between  $\bar{L}f(\hat{K}/L) - \hat{K}$  and  $\bar{L}f(\bar{K}/L) - \bar{K}$ .

The rate of inflation is equal to the rate of monetary expansion. Note that the model is quite consistent with the rate-of-return dominance of capital over money in the steady state. The rate of return of the former is  $(1/\rho) - 1$ , while that of the latter is  $-m/(1 + m)$ .

By (24), the ratio of consumption on the date of financial transactions to consumption in the following period is  $(1 + m)/\rho$ ; it rises as inflation rises. Therefore, inflation distorts the intertemporal consumption decisions. It leads people to consume more right after they withdraw money and less in later periods.<sup>8</sup> The rate of deflation, which is such that intertemporal consumption decisions are optimal from the point of view of society, is  $1 - \rho$ . As Friedman (1969) proposed, the rate of deflation must be equal to the discount rate. This result was also obtained by Jovanovic (1982) in a model in which money is held for transactions purposes but in which people, while picking the timing of their visits to the financial intermediaries optimally, do not pick their consumption path optimally. Here it can be shown as follows. Consider a social planner who wants to maximize

$$\sum_{\tau=1}^{\infty} \rho^{\tau-1} [\alpha \ln C_{\tau}^{\tau} + (1 - \alpha) \ln C_{\tau}^{\tau-1}] \quad (26)$$

subject to

$$K_{\tau} = \bar{L}f\left(\frac{K_{\tau-1}}{\bar{L}}\right) - n(C_{\tau}^{\tau} + C_{\tau}^{\tau-1}) - nB, \quad (27)$$

where  $\alpha$  is a weight between zero and one. This social planner maximizes a convex combination of the utilities of both types of consumers subject to society's budget constraint. The plan that maximizes (26) satisfies  $C_{\tau}^{\tau} = [\alpha/(1 - \alpha)]C_{\tau}^{\tau-1}$  and

$$C_{\tau+1}^{\tau} = \rho f'\left(\frac{K_{\tau-1}}{\bar{L}}\right) C_{\tau}^{\tau}. \quad (28)$$

In the steady state in which  $\rho f'(K_{\tau}/\bar{L})$  is equal to one, consumption is constant. Indeed, this is precisely what occurs in the decentralized economy with money as long as, in (7),  $\rho P_t/P_{t+1}$  is equal to one. This in turn requires that  $m$  be equal to  $(\rho - 1)$  as claimed.

However, there is a problem in sustaining the equilibrium with  $K = \bar{K}$  when  $m$  is equal to  $(\rho - 1)$ . This problem arises because at this equilibrium money and capital have the same rate of return. Therefore households would prefer to withdraw money at the beginning of

<sup>8</sup> This has also been noted by Jovanovic (1982). This effect is likely to be even more pronounced on consumer expenditure when there are durable goods, and consumer expenditure can be different from consumption. In this paper, consumption and consumer expenditure coincide by assumption.

their lives in the amount equal to the present discounted value of their income. Then they would avoid all visits to their bank and associated transactions costs. This would result in no capital being available in this model with 100 percent reserves. It may well be the case that, with a smaller reserve requirement, the equilibrium would be sustainable. In any event, note that, as long as  $m$  is just slightly bigger than  $(\rho - 1)$ , the equilibrium of this model is essentially equal to the Pareto optimum and does induce people to hold  $\bar{K}$  units of capital.

Inflation induces people to consume less in the period in which they do not go to the bank. Therefore,  $M_\tau/P_{\tau+1}$ , which is equal to this consumption, falls. However, surprisingly, in this model  $M_\tau/P_\tau$  actually rises with inflation. This result is undoubtedly due in part to the fact that people do not go to the bank more often when inflation rises. It emerges because, even though people want to reduce  $M_\tau/P_{\tau+1}$ , they must increase  $M_\tau/P_\tau$  to ensure that inflation does not reduce  $M_\tau/P_{\tau+1}$  too much. The result can be established by noting that equation (18) says that  $M_\tau/P_\tau$  is proportional to consumption in the period in which people visit the financial intermediary. By equation (19), this consumption does indeed rise with inflation.

## V. The Nonneutrality of Monetary Policy

The main purpose of this paper is to study conditions outside the steady state. First, it will be established that a wide variety of monetary policies (or open-market operations) affect aggregate output. This is the consensus of textbooks, such as Branson (1979). However, this view has recently been challenged by a variety of authors (e.g., Wallace 1981; Chamley and Polemarchakis 1983). These authors have shown that in models in which money is held only for its rate of return characteristics, open-market operations are neutral. Admittedly the premise of these models—that money is not rate of return dominated by other assets—appears to be a bad description of free-market economies as we know them. The proof that open-market operations can affect output in the economy of this paper is straightforward. Consider a base path for money and taxes,  $\{M_\tau\}$  and  $\{T_\tau\}$ ; then the equilibrium sequence of capital  $\{\bar{K}_\tau\}$  must satisfy (20).

Now consider a slightly different financial policy for the government. At  $t$ , unexpectedly, the government purchases some extra capital by issuing  $\epsilon$  units of money. Then, at  $T$ , the government engages in the reverse transaction: it sells  $\epsilon[\prod_{\tau=t}^T f'(K_\tau/\bar{L})]/P_t$  units of capital. Therefore, the path of taxes remains unchanged, but between  $t$  and  $T$  the path of money is replaced by  $\{M_\tau + \epsilon\}$ . After  $T$ , the path of money is given by  $\{\tilde{M}_\tau + \epsilon[1 - P_T \prod_{i=t}^T f'(K_i/\bar{L})]/P_t\}$ . Then, it is clear from (20) that if, in the new equilibrium, capital remains unchanged from

$\{\bar{K}_\tau\}$  at  $t$  and  $t + 1$ , it will have to be different from  $\bar{K}_{t+2}$  at  $t + 2$ ; monetary policy affects output.

The intuition behind this result is as follows. Suppose the open-market operation had no effect on prices. Then the paths of consumption would be unaffected. However, the people who visit the bank at  $t$  would not want to demand the increased amount of money balances. If, instead, any rate of inflation after  $t$  were affected by the open-market operation, then by (7) some individuals would change their consumption path. Finally, suppose that only the price at  $t$  was affected by the government's change of financial policy. Then the consumption of those who visited the bank at  $t - 1$ , which is given by  $M_{t-1}/P_t$ , would be affected. So the nonneutrality of open-market operations hinges crucially on the fact that not everyone visits the bank on the day of the operation.

I am not just interested in establishing that monetary policy is nonneutral. Instead I want to characterize its effect. To do this for general policies, it would be necessary to solve the nonlinear difference equation (20). Two simpler approaches are pursued here. The first is to simulate the unique nonexplosive path given by equation (20) and the initial condition (23). This nonexplosive path is obtained numerically by assuming that  $\bar{L}$  is equal to one, the production function is given by  $Q_t = K_t^{.75}$ , and  $\rho$  is equal to .99. The steady state of this economy has a capital stock of .30394. I assume that in period 0 the economy is at this steady state. Therefore the difference in the holdings of capital of the two types of people at zero,  $n(K_0^2 - K_0^1)$ , is given by  $\rho[\bar{K} - f(\bar{K})]/(1 + \rho)$ . Figure 1 presents the path of capital that ensues from a 2 percent increase in the money stock that occurs unexpectedly in period 1. This path is obtained by searching over different values of  $K_1$ . For each value of  $K_1$ , (23) is used to obtain  $K_2$  and (20) is used to obtain the latter levels of capital. Values of  $K_1$  slightly lower than those reported lead to implosions in capital, while slightly higher ones lead to explosions. Interestingly, for the parameters I considered, the system does not tend to oscillate. Figure 1 shows that the monetary expansion increases capital, albeit by a small amount. Capital then slowly returns to its steady-state level. The intuition behind these results is the following. When the quantity of money is increased at  $\tau$ , the price level rises. This decreases  $M_{\tau-1}/P_\tau$  and therefore reduces  $C_{\tau-1}^\tau$ . This fall in the consumption of those who do not visit their bank at  $\tau$  raises capital and hence output in the following period. This explanation suggests that monetary policy may derive much of its power in this model from the assumption that those people who visited the bank at  $\tau - 1$  do not change their pattern of bank visits in response to inflation at  $\tau$ . How much people who had

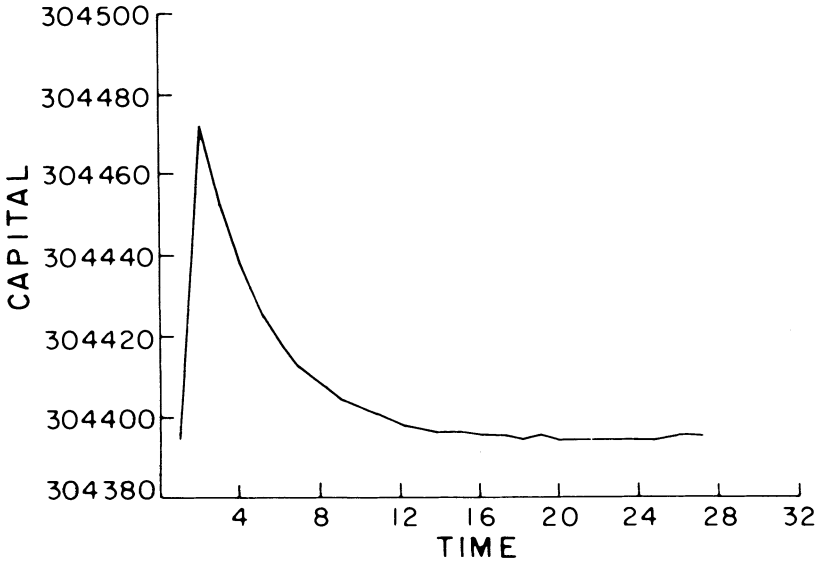


Fig. 1.—Open-market monetary increase in period 2

not scheduled a visit to their intermediary at  $\tau$  would reduce their consumption in response to inflation at  $\tau$  if they were free to pick the timing of these visits optimally is an open question that deserves further research.

Figure 2 shows the effects on the path of capital of a 2 percent increase in the money supply in period 5 that is announced in period 1. Capital and output immediately begin to fall and continue to fall until period 5. Then they both jump up. This jump is not significantly smaller than the one produced by an unexpected monetary expansion. The low level of capital in the period just preceding the monetary expansion can be explained as follows. The monetary expansion leads people in period 4 to expect high inflation. Therefore, the households that visit the intermediaries in period 4 consume a disproportionate amount at 4 and thus lower the capital stock. The fall in the capital stock the moment the future monetary increase is announced has an unexpected consequence. It means that output increases more when an unexpected monetary injection's effect on government capital is expected to be neutralized by a future open-market operation than when it is expected to be neutralized by giving equal lump-sum transfers to the entire population.

The second approach is to compute the solution using the linearized versions of (20) and (23) around the steady state. Assuming that  $\bar{L} = 1$  and that  $g_{t+3}$ , which is given by  $[f(\bar{K}) - \bar{K}]\{1 + \rho(M_{\tau+2}/$

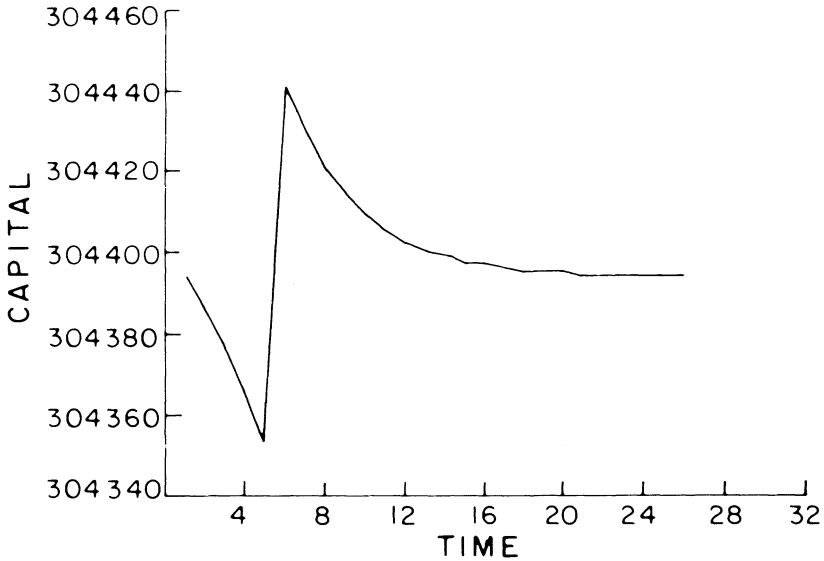


FIG. 2.—Open-market monetary increase in period 5

$M_{\tau+3})/[1 + \rho(M_{\tau}/M_{\tau+1})] - 1\}$ , is close to zero, the linearized version of (20) becomes

$$(1 + L)(1 - \lambda_2 L)(1 - \lambda_3 L)(K_{\tau+3} - \bar{K}) = -g_{\tau+3}. \quad (29)$$

The nonexplosive solution of (29) is given by

$$(K_{\tau} - \bar{K}) = \lambda_3(K_{\tau-1} - \bar{K}) + A(-1)^{\tau} + \frac{1}{1 + \lambda_2} \sum_{j=0}^{\infty} \left[ \left( \frac{1}{\lambda_2} \right)^j - (-1)^j \right] g_{\tau+j+1}, \quad (30)$$

where  $A$  must be picked to satisfy the version of (23) that is linearized at the steady state<sup>9</sup>

$$(K_{t+1} - \bar{K}) - \{2f'(\bar{K}) - \rho f''(\bar{K})[f(\bar{K}) - \bar{K}]\}(K_t - \bar{K}) + \{f'(K)^2 - (1 - \rho)f''(\bar{K})[f(\bar{K}) - \bar{K}]\}(K_{t-1} - \bar{K}) = -f'(\bar{K})g_{t+2}. \quad (31)$$

Using (30) for  $t$  and  $t + 1$ , assuming  $t$  is even, and substituting in (31), one obtains  $A$ :

<sup>9</sup> For simplicity, (31) ignores money growth between  $t$  and  $t + 2$ .

$$\begin{aligned}
 A = & \left\{ \frac{\lambda_3 + f'(\bar{K})^2 - (1 - \rho)f''(\bar{K})[f(\bar{K}) - \bar{K}]}{B} - \lambda_3 \right\} (K_{t-1} - \bar{K}) \\
 & + \left[ \frac{f'(\bar{K})}{B} + \frac{1}{\lambda_2 B} - \frac{1}{\lambda_2} \right] g_{t+2} + \frac{1}{1 + \lambda_2} \\
 & \times \left[ \sum_{j+1}^{\infty} \left( \frac{1}{\lambda_2} \right)^j \left( \frac{1}{B} + \frac{1}{\lambda_2 B} - \frac{1}{\lambda_2} \right) g_{t+j+2} - \sum_{j+1}^{\infty} (-1)^j g_{t+j+2} \right], \tag{32}
 \end{aligned}$$

$$B = 1 - \lambda_3 + 2f'(\bar{K}) - \rho f''(\bar{K})[f(\bar{K}) - \bar{K}].$$

An unexpected increase in money at  $t$  raises  $g_{t+2}$  and leaves all  $g_{\tau}$ 's for  $\tau$  greater than  $t + 2$  unaffected. So, it raises  $K_t$  by  $[f'(\bar{K}) + (1/\lambda_1)]/B$ . Assuming the same technology and monetary shock that lead to figure 1, the change in  $K_t$  is .0007838. This is very similar to the change of .000778 computed to arrive at figure 1. However, since, in the linear approximation,  $A$  is equal to .0000062, the linear approximation exhibits small oscillations in the steady state that are absent in the nonlinear system. If the linear approximation is forced to converge by choosing  $A$  equal to zero, the change in capital at  $t$  is .0007826. So, even though the presence of the unit root implies that the linearized system is not, as a general rule, a good approximation to the behavior of the nonlinear system, it appears to give reasonable answers in the context studied here.

## VI. Conclusions

The model of this paper is a modest step toward the construction of tractable and realistic general equilibrium models capable of shedding light on the effects of non-steady-state monetary changes. Its major advantage is that people's motive for holding money is explicitly that money is used for transactions. In particular, in those periods in which households do not visit their banks, they are faced with an extreme version of the "Clower constraint": they must pay for their purchases with money carried over from the previous period. This ensures that monetary policies that expand money and prices reduce the real consumption of those households that do not visit their financial intermediary on the day of the monetary expansion. This fall in consumption raises capital and output in future periods.

A number of issues are raised by this paper. First, an important question is to what extent the power of monetary policy would be diluted if people timed their visits to intermediaries optimally. Associated with this question is the empirical question of whether people in

fact do significantly alter the interval during which they refrain from visiting their bank as events change.

It is hoped that the framework of this paper can also be used to study the effects of various institutional setups on macroeconomic activity. In particular, it should be capable of shedding some light on the difference between commodity standards, fractional reserve standards, and the 100 percent reserves standard of this paper.

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