Monopolistic Price Adjustment and Aggregate Output

Julio J. Rotemberg


Stable URL:
http://links.jstor.org/sici?sici=0034-6527%28198210%2949%3A4%3C517%3AMPAAAO%3E2.0.CO%3B2-9

*The Review of Economic Studies* is currently published by The Review of Economic Studies Ltd.
Monopolistic Price Adjustment and Aggregate Output

JULIO J. ROTEMBERG
Massachusetts Institute of Technology

This paper studies the consequences for the behaviour of aggregate output of the perception on the part of firms that changing prices is costly. The rational expectations equilibrium of an economy with many such firms is constructed. It is shown that in this economy nominal shocks have a persistent effect on aggregate output. Furthermore, the real wage is demonstrated to move procyclically in such an economy.

1. INTRODUCTION

There have been two major attempts to build a microeconomic foundation for the existence of fluctuations in aggregate output. The first one is associated with the work of Barro and Grossman (1976) in which prices are assumed to be fixed or else to follow some slow path towards their equilibrium values. The economic agents then proceed to maximize their objective functions subject to the fixed prices and to the rationing that naturally emerges in those markets in which supply is not equal to demand at the going prices. The second attempt is associated with the seminal papers by Lucas (1972, 1975). He built an equilibrium model of the business cycle. In his model the prices are such that people succeed in carrying out the transactions they desire to carry out at these prices. However, agents are assumed to misperceive profitable opportunities. This is due to their inability to observe the value of aggregate statistics like the current levels of prices and of the money supply. Instead monetary injections are momentarily perceived as good opportunities by everybody. Therefore they are followed by increases in aggregate output which dissipate as people learn about the old monetary injections.

This paper presents a new attempt at building a model which accounts for the existence of fluctuations in aggregate output in response to nominal disturbances, like an unpredicted injection of money into the economy. Like Lucas' model, it is an equilibrium model, albeit not a competitive one. Economic agents maximize their objective functions taking the prices set by the other agents as given. They make the best use of current information in the computation of facts about their current and future economic environment. In fact, the producers, who in this model produce differentiated products, have full information about the present. Namely, they know the prices charged by their suppliers, the price level, and the economy-wide level of nominal money balances. Furthermore, they observe their demand and cost functions before they set their prices. These assumptions about the information available to producers sets this model apart from Lucas' and in my view constitute a theoretical advantage.

In this model it is the assumption that it costs resources to change prices, possibly due to the difficulties changing prices impose on consumers, that introduced the "rigidity" necessary for the existence of correlated responses in output to uncorrelated nominal shocks. The model is therefore a relative of the Barro-Grossman model in that it is the slow response of prices which is placed at the centre of the explanation of business cycles. However, it parts company with the latter model in that the monopolies set their prices
optimal, given that it is costly to change them. This fact changes many of the qualitative features of the Barro–Grossman model.

The model of this paper has the added advantage of implying a positive correlation between the real wage and GNP. Instead of both the standard Keynesian and most variants of the Lucas model require the real wage to be relatively low when the output is relatively high. In the U.S. detrended real hourly earnings seem to be positively correlated with detrended output (see for instance Dunlop (1938), Tarshis (1939) and Blinder (1980)).

In the next section I discuss a monopolist’s optimization problem when he is faced by specific demand and cost functions and when changing prices is costless. In Section 3 I construct the equilibrium of an economy with many such monopolists. In Section 4, I first derive the pricing rule for a monopolist when changing prices is costly. Then, I construct the rational expectations equilibrium for the economy with costly price adjustment. This rational expectations equilibrium is a stochastic process for the price level and for output that is driven by the stochastic shocks to the level of money balances and to the taste for real money balances. In Section 5, I study some of the comparative dynamics of this rational expectations equilibrium. I show that when the money supply follows a random walk, output will be serially correlated. Furthermore I study the behaviour of the equilibrium when there is a constant expected rate of monetary expansion and, alternatively, when the money stock is expected to change drastically in the future. In Section 6, I extend the model to include a labour market and show that real wages move procyclically. The last section contains some conclusions and a research agenda.

2. MONOPOLIES

The model consists of \( n \) monopolies indexed by \( i \). Each one produces a distinct non-storable good. In this section I derive the pricing rule that the monopolists will follow in the absence of a cost to changing prices.

Both consumers and firms demand good \( i \). The demand functions are assumed to be given by:

\[
Q^d_{it} = A_i \left( \frac{P_i}{P_t} \right)^{-b_i} \left( \frac{M_t}{P_t V_t} \right)^{d_i}, \quad i = 1, 2, \ldots, n
\]  

(1)

where \( Q^d_{it} \) is the quantity of good \( i \) demanded at time \( t \); \( A_i, d_i \) and \( b_i \) are firm specific constants; \( P_i \) is the price of good \( i \) at time \( t \) which is set by the monopolist; \( P_t \) is the price level which is defined below; \( M_t \) is the economy-wide level of nominal money balances and \( V_t \) is a time varying taste parameter. Prices will change because \( V_t \) and \( M_t \) follow stochastic processes with normal disturbances. The monopolists are assumed to observe their demand function at time \( t \). Moreover, the money supply is assumed to be a published statistic at time \( t \). Therefore the monopolists observe both \( M \) and \( V \) in the current period.

The quantity demanded of any particular good depends not only on the relative price of the good, but also on real money balances. Such demand functions will arise if real money balances are a direct source of utility. They may also arise when money is just a store of value.¹ Note that as \( V \) changes a different level of real money balances leads to the same quantity of each good being demanded at constant relative prices. The second term of these demand functions can be thought of as a wealth effect.

The price level is a weighted geometric average of the \( n \) prices charged by the monopolists:

\[
P_t = \left[ \prod_{i=1}^{n} (P_i)^{h_i} \right]^{1/\Sigma h_i}
\]  

(2)

where the \( h_i \) are constants. In Section 4, labour becomes the only factor of production. For simplicity I start, however, considering an economy in which only goods are required
to produce goods. Let the production functions of the monopolists be given by:

\[ Q_{it} = \prod_{j=1}^{n} F_{ij}^{(h_{i}/2, h_{i})} \quad i = 1, \ldots, n, \]

where \( F_{ij} \) is the quantity of good \( j \) used in the production of good \( i \) at \( t \). All goods are used in the production of good \( i \). Moreover, the production functions (3) exhibit decreasing returns to scale. Hence, given a feasible vector of outputs, the vector of outputs obtained by multiplying the given vector by a sufficiently large scalar is not feasible. Such an expansion requires that the inputs be multiplied by \( \mu^2 \) when the outputs are multiplied by \( \mu \). For \( \mu \) sufficiently big the inputs must be larger than the outputs. Therefore, there is a production possibility frontier.

I will assume that when purchasing its own output for use as an input, the productive side of the firm charges a price which is equal to the price the firm charges its other customers times a fixed discount factor \( \psi \). Then the productive side of the firm proceeds to minimize costs. This assumption would not be necessary if good \( i \) were not productive in the production of good \( i \). As long as the weight of any given good is small in the price level this assumption will not have any important consequences. It also appears to be of descriptive value. Then, the cost functions of the monopolists are:

\[ C_i(Q_{it}) = (UP_{it}Q_{it})^{2} / 2 \quad i = 1, \ldots, n \]

\[ U_i = \psi^{h_i/\Sigma h_i} \prod_{i=1}^{n} \left( \frac{2 \Sigma h_i}{h_i} \right)^{h_i/\Sigma h_i} \]

where \( C_i(Q_{it}) \) is the cost to firm \( i \) of producing the quantity \( Q \) at time \( t \). The monopolists take every other price as given. Moreover, I will assume that \( h_i/\Sigma h_i \) is very small and that they can neglect their influence on the price level. Then, the maximization of profits requires that \( P_{it} = P^*_i \) where:

\[ P^*_i = \Lambda_i U_i P_{it} Q_{it} \quad i = 1, 2, \ldots, n, \]

\[ \Lambda_i = b_i / (b_i - 1) \]

and \( b_i \) must be larger than one for the monopolist to be at a profit maximum.

3. EQUILIBRIUM WITHOUT COSTS OF CHANGING PRICES

In this section I construct the equilibrium of an economy consisting solely of monopolists and of the households that own the monopolies and spend the profits on output. This equilibrium is constructed in two subsections.

In the first I discuss the form of Walras Law in this economy. I the second subsection I construct the equilibrium that would prevail in the absence of costs to changing prices. This equilibrium is constructed for two reasons. First, this will be the appropriate "long-run" equilibrium concept when changing prices is costly. Second, I will show that this equilibrium is strictly inside the production possibility frontier and that allocations with more output (which will emerge when changing prices becomes costly) are feasible.

3.A. Walras law

The output of the monopolists goes to two sectors. Some is demanded by other firms as an input, and some is demanded by households. Households are given the profits of the monopolists and it is these that they use to purchase goods. Let \( D_{it} \) be the demand by households of good \( i \) at time \( t \). The households also have a demand for the \( n+1 \)st good, money. This demand is written as a demand for flows of additional amounts of nominal money above the previous stock of money:

\[ M^d_t - M_{t-1} = M^b_t - M_{t-1} + \sum_i \Pi_i - \sum_t P_{it} D_{it} = Y_t - C^d_t. \]
Here $Y_i$ is nominal income of households in period $t$, $\Pi_{it}$ are the profits of firm $i$ and $t$ and $C^{d}_it$ is the amount of consumption demanded in period $t$. The increase in nominal money supplied is part of the income of the households since here money is assumed to be distributed by a helicopter and held only by households from one period to the next.

Equation (6) when the desired quantities are replaced by the actual quantities transacted is also the households budget constraint. Since there is no rationing, the ex ante equation (6) and the ex post budget identity are the same.

The individual firm’s budget constraint is:

$$\Pi_{it} = P_i Q_{it} - \sum_i P_i F_{it} = \Pi_{it} = P_i Q_{it} - \sum_i P_i F_{it} = i = 1, 2, \ldots, n.$$

(7)

The equilibrium condition in the goods market is that:

$$Q_{it} = D_{it} + F_{it} = D_{it} + \sum_i F_{it} = i = 1, 2, \ldots, n.$$  

(8)

Replacing (7) and (8) and adding over all firms:

$$\sum_i \Pi_{it} = \sum_i P_i Q_{it} - \sum_i \sum_i P_i F_{it} = \sum_i P_i D_{it}.$$  

(9)

Therefore equilibrium in the $n$ goods markets implies equilibrium in the money market. This can be seen by replacing for total profits in equation (6). I will therefore consider only the goods markets when computing the equilibrium of this economy.

3.B. Equilibrium

Taking logarithms on both sides of (4):

$$p^{\Pi}_{it} = p_i + (\lambda_i + u_i + a_i)/(1 + h_i) + d_i(m_i - p_i - v_i)/(1 + b_i)$$

(10)

where lower case letters denote the logarithms of their respective upper case letters.

The equilibrium price level can be computed by weighting the above equations by $h_i$ and summing:

$$\sum h_i p^{\Pi}_{it} = p_i \sum h_i + \sum_i (\lambda_i + u_i + a_i)/1 + h_i] = \sum_i d_i h_i / [1 + b_i](m_i - p_i - v_i).$$

(11)

Replacing (2) in (11):

$$m_i - p_i - v_i = - \left[ \sum_i h_i (\lambda_i + u_i + a_i)/1 + h_i \right] / \left[ \sum_i d_i h_i / 1 + b_i \right].$$

(12)

Equation (12) says that real money balances depend on all the parameters of the model. It also can be used to see whether an expansion in the quantity of money $m$, or a change in the taste for real money balances $v_i$ has any effect on real output.

I will use as an index of aggregate output the sum of the logarithms of the output of the $n$ goods.

$$q_i = \sum_i q_{it}$$

(13)

And using (1) in the definition of (28) output demanded is:

$$q_i = \sum_i a_i + (m_i - v_i - p_i) \sum_i d_i$$

(14)

The equilibrium condition (12) assures that $m_i - p_i - v_i$ is unaffected by changes in either the level of money balances or the taste for real money balances. Therefore, by (14) aggregate output will not respond to such changes.

This economy is neutral. Increases in nominal money balances are matched by increases in the price level. The “real” side of the economy, namely outputs and relative prices, remains unchanged. The neutrality of this economy depends on both the monopolists’ knowledge of their input prices, demand and cost functions, and on the absence of any costs to changing prices.

The competitive equilibrium can be computed in exactly the same way except that $A_i$ is in this case equal to one. That is competitors set price equal to marginal cost. Then
\( \lambda_i \) is zero and the weighted sum of the \( \lambda_i \) that appears in (12) is zero. On the contrary, the \( \lambda_i \)'s must be positive when the economy consists of monopolists. Therefore real money balances are larger when the economy is competitive. Larger real money balances lead to a larger aggregate output by (14). The monopolists produce less than the competitors in equilibrium. Recall that the competitive equilibrium is on the production possibility frontier. Therefore the monopolistic equilibrium without costs to changing prices is strictly inside the production possibility frontier. This means that allocations with more output than the one corresponding to this static equilibrium are feasible. When analyzing the behaviour of the economy with costly price adjustment it will be apparent that sometimes the price level will be below the price level that would prevail in the absence of such costs and at other times it will be above. As long as the price level is not too far below the price level the monopolists would desire, the allocation will be technologically feasible.

4. EQUILIBRIUM WITH COSTLY PRICE ADJUSTMENT

In this section, I first find the pricing rule each monopolist will employ when price changes are costly. Then I construct the equilibrium that emerges when each monopolist has rational expectations and takes the paths of prices he expects other firms to charge as given.

4.A. The pricing rule

To compute the pricing rule when price changes are costly, I first need to approximate the difference between revenues from sales and costs of production by a quadratic function of \( p_u \). Real profits of firm \( i \) at \( t \) are given by

\[
\pi\left( \frac{P_u}{P_t}, \frac{M_t}{P_t}, \frac{V_t}{P_t} \right) = A_t\left( \frac{P_u}{P_t} \right)^{1-b_i} \left( \frac{M_t}{P_t} \right)^d \left[ A_t\left( \frac{P_u}{P_t} \right)^{-b_i} \left( \frac{M_t}{P_t} \right)^{d-2} \right] \\
= \pi\left( \frac{P_u^*}{P_t}, \frac{M_t}{P_t} \right) + \frac{\partial \pi(P_u^*/P_t, M_t/(P_i, V_t))}{\partial(P_u/P_t)} \left( \frac{P_u}{P_t} - \frac{P_u^*}{P_t} \right) \\
+ \frac{1}{2} \frac{\partial^2 \pi(P_u^*/P_t, M_t/(P_i, V_t))}{\partial(P_u/P_t)^2} \left( \frac{P_u}{P_t} - \frac{P_u^*}{P_t} \right)^2
\]

where \( P_u^* \) is given by (10). The first order term vanishes by the definition of \( P_u^* \). Hence profits can be written as:

\[
\pi\left( \frac{P_u}{P_t}, \frac{M_t}{P_t}, \frac{V_t}{P_t} \right) = \pi\left( \frac{P_u^*}{P_t}, \frac{M_t}{P_t}, \frac{V_t}{P_t} \right) \\
+ \frac{1}{2} \left( 1 - b_i \right)^3 \left( A_t U \frac{b_i}{b_i - 1} \right)^{1/(1 + b_i)} \left( \frac{M_t}{P_t} \right)^{2d/(1 + b_i)} \left( P_u - P_u^* \right)^2
\]

where (5) was used to substitute for \( P_u^*/P_t \). This expression can be further approximated by:

\[
\pi\left( \frac{P_u}{P_t}, \frac{M_t}{P_t}, \frac{V_t}{P_t} \right) = \pi\left( \frac{P_u^*}{P_t}, \frac{M_t}{P_t}, \frac{V_t}{P_t} \right) \\
+ \frac{1}{2} \left( 1 - b_i \right)^3 \left( A_t U b_i \frac{b_i}{b_i - 1} \right)^{1/(1 + b_i)} \left( P_u - P_u^* \right)^2 \\
+ \frac{d_i(1 - b_i)}{(1 + b_i)(b_i - 1)} \left( A_t U \frac{b_i}{b_i - 1} \right)^{1/(1 + b_i)} Z^{2d_i/(1 + b_i)} \left( P_u - P_u^* \right) \left( M_t - p_t - v_t - z^* \right)
\]
where $Z^*_t$ is the value of $M_t/P_tV_t$ at the equilibrium without costs of changing prices (12). By (11) $(m_t - p_t - u_t - z^t)$ is of the same order of magnitude as $(p_t - p^*_t)$. Hence the last term in (15) is of third order in $(p_t - p^*_t)$ and can be neglected as long as $p_t$ is not too far away from $p^*_t$.

Equation (15) gives the profits that would be forthcoming if changing prices were costless. The monopolists will now be assumed to be concerned with the maximization of the discounted value of the above profits minus the cost of price changes.

As has often been pointed out (Barro (1972), Mussa (1976), Sheshinski and Weiss (1977)), changing prices is costly for two reasons: First, there is the administrative cost of changing the price lists, informing dealers, etc. Secondly, there is the implicit cost that results from the unfavourable reaction of customers to large price changes.

While the administrative cost is a fixed cost per price change, the second cost can be a different function of the magnitude of the price change. In particular, customers may well prefer small and recurrent price changes to occasional large ones. This is what is implicitly assumed in this paper as I make the costs to changing prices a function of the square of the price change. The modeling of the characteristics of consumers which leads them to these preferences is an important undertaking which is beyond the scope of this paper. Such modeling would make the costs of changing prices endogenous.

The assumption that price changes are perceived to be costly is not only plausible on theoretical grounds but can also be justified by its power to explain the pricing behaviour of manufacturing firms. In empirical studies of these prices (Eckstein and Wyss (1972)), lagged prices are important explanators of current prices even when all those variables which explain “desired” prices are included in the regressions.

When changing prices is costly, the monopolist might consider maintaining the old price while rationing some customers. This is ruled out in this model by implicitly assuming that the costs of rationing customers are enormous. These costs, too, are related to the reputation of the individual monopolists.

The existence of costs to changing prices alters the qualitative form of the monopolist's maximization problem. In the absence of these costs he chooses his prices in each period according to (5). In their presence, today's decisions (prices) affect tomorrow's profits. This is so because it will be costly to change tomorrow a price different from the one the firm decides to change today. Therefore at time $t$ the best price for $t$ must be determined by maximizing the expected present discounted value of real profit given by:

$$E_t(\sum_{k=0}^{\infty} \rho^k [(p^*_t + k - w_t (p_t + k - p^*_t + k)^2 - c_i (p_t + k - p_t + k - 1)^2] \tag{16}$$

where

$$w_t = \frac{(h_t - 1)^3}{2b_tU} \left( \frac{A_tU h_t}{b_t - 1} \right)^{1/(1 + h_t)} Z^{*2d/(1 + h_t)},$$

$c_i$ is a firm specific constant and $\rho$ is the firm's discount factor. I now impose a relation between $c_i$ and the other firm specific parameters ($A_i, b_i$ and $d_i$) by assuming that:

$$c_i/w_i = c, \quad i = 1, \ldots, n \tag{17}$$

where $c$ is a constant. This assumption is required to make the equilibrium of this economy tractable.

The monopolist's problem can now be rewritten as:

$$\min E_t(\sum_{k=0}^{\infty} \rho^k [(p_t + k - p^*_t + k)^2 + c(p_t + k - p_t + k - 1)^2]) \tag{18}$$

(18) is a standard optimal control problem that satisfies the conditions for first period certainty equivalence. Problems of the form of (18) have been solved by numerous authors. See Kennan (1979) for an extensive list of references from the investment
literature. Sargent (1979) also solves problems of this type. His method of solution is employed here.

What is conceptually different about equation (18) is that here it is costly to adjust prices. Instead, the literature on costs of adjustment has concerned itself with competitive firms that take prices as given and have costs to changing quantities (employment, the capital stock, etc.).

The minimization of (18) leads to the following first order conditions:

\[
\left[1 - (1/\rho c + 1/\rho + 1)L + \frac{1}{\rho}L^2\right]p^i_{u/t+k} = \frac{L}{\rho c} p^*_{u/t+k}
\]  
(19)

where the superscript \(i\) indicates that these are expectations formed by firm \(i\). \(p^i_{u/t+k}\) is the expectation formed by firm \(i\) at \(t\) of the price \(p_{u+k}\) while \(p^*_{u/t+k}\) is the expectation formed by firm \(i\) at \(t\) of the price \(p^*_{u+k}\).

The minimization of (18) also involves a transversality condition:

\[
\lim_{k \to \infty} (p^i_{u/t+k} - p^*_{u/i+k}) + c(p^i_{u/t+k} - p^i_{u/t+k-1}) = 0.
\]  
(20)

Factoring the expression on the LHS of (13)

\[
(1 - \alpha L)(1 - \beta L)p^i_{u/t+k} = \frac{L}{\rho c} p^*_{u/t+k}
\]  
(21)

where

\[
\alpha + \beta = 1/\rho c + 1/\rho + 1
\]

\[
\alpha \beta = 1/\rho
\]

and therefore

\[
(1 - \alpha)(1 - \beta) = -\frac{1}{\rho c}.
\]  
(22)

Since the sum and the product of the two roots are both positive, the roots are positive. Furthermore since both \(\rho\) and \(c\) are positive, (22) implies that one of the roots is greater than one while the other is smaller than one.

This can be seen in Figure 1. The intersections of the two curves give the roots \(\alpha\) and \(\beta\).

![Figure 1](image-url)
Let $\alpha$ be the smallest root. In Figure 1 one can see that a decrease in $c$ moves the straight line to the right thereby increasing $\beta$ and reducing $\alpha$. Instead an increase in $\rho$ has ambiguous effects since it shifts both curves to the right.

To obtain the values of the sequence $p_{i,t+k}$ from (20) two endpoint conditions are needed. One initial condition is the value of $p_{i,t-1}$ which is given to the firm by its own history. The other endpoint condition is given by (19). This transversality condition says that the firm expects to charge a price close to $p_{i,t+k}$ in the far future. It requires, as shown in Sargent (1979) that the “unstable” root $\beta$ be used to solve for $p_{i,t+k}$ forwards in time. In other words, it will be satisfied as long as both sides of (14) are divided by $(1-\beta L)$ yielding:

$$p_{i,t+k} = \alpha p_{i,t+k-1} + \frac{1}{\beta \rho c} \sum_{j=0}^{\infty} \left(\frac{1}{\beta}\right)^j p_{i,t+k+j}.$$  

(23)

This equation gives the path of prices the firm expects to charge from $t$ on as a function of its expectations at $t$ of the sequence $p_{i,t+j}^*$, where these expectations are taken as given. It then charges $p_{i,t}^*$ at $t$. At $t+1$ it will revise its expectations about $p_{i,t+1}^*$, the prices that will be charged at $(t+1)$ can be computed as in (23) by replacing $k$ by 1. In general the equation that determines the price charged by firm $i$ at $t$ is:

$$p_{i,t} = \alpha p_{i,t-1} + \frac{1}{\beta \rho c} \sum_{j=0}^{\infty} \left(\frac{1}{\beta}\right)^j p_{i,t+j}.$$  

(24)

The existence of a cost to changing prices makes the monopolist change his prices slowly (therefore prices at $t$ depend on prices at $t-1$). These costs also lead monopolists to take into account the expected future optimal price when deciding on the current price so as to avoid costs of changing prices in the future. The price that the $i$th monopolist charges today is a weighted average of the price he charged yesterday and the prices he would like to charge in all future periods if there were no cost to changing prices. Equation (25) is very similar to Sargent’s (1979) characterization of labour demand when adjusting employment is costly. There, employment today depends on employment yesterday and the expected future path of real wages.

Notice that if $p_{i,t}^*$ follows a random walk its expectations at $t$ for the infinite future is just its $p_{i,t}^*$ and equation (24) reduces to:

$$p_{i,t} = \alpha p_{i,t-1} + \frac{1}{\beta \rho c} \frac{\beta}{\beta - 1} p_{i,t}^* = \alpha p_{i,t-1} + (1-\alpha) p_{i,t}^*.$$  

(25)

The last equality is obtained by using (22). Equation (25) is just the partial adjustment equation often used in empirical research. An increase in $c$, the cost to changing prices, was shown to increase $\alpha$. Thus, such an increase slows the adjustment of prices towards their long-run value, as expected. On the other hand, a change in the discount rate has an ambiguous effect on this speed of adjustment. While a decrease in $\rho$ makes it relatively cheaper to change prices in the future (thereby leading to slower adjustment), it also penalizes the monopolist relatively more for current deviations of $p_{i,t}$ from $p_{i,t}^*$.

4.B. Equilibrium

In this subsection the rational expectations equilibrium of an economy in which each firm expects to follow (23) is constructed. Each firm takes the expected path of $p_{i,t}^*$ as given. Therefore firms do not expect to influence the paths of $m_{i,t}$, $p_{i,t}$, and $v_{i,t}$. However, the path of $p_{i,t}^*$ they expect it is the mathematical expectation of $p_{i,t}^*$ conditional on each firm picking its price according to (24). This equilibrium could therefore be called a Nash-Rational Expectations Equilibrium. It has the property that the prices charged by each firm at $t$ are optimal given the prices charged by other firms, the mathematical
expectation of the prices other firms will charge in the future and given that firms have
to sell the quantities demanded at these prices.

Denote the expectation held at $t$ by firm $i$ of the values at $t+j$ of the price level,
the level of money balances and $\nu$ by $p_{t+i+j}$ $m_{t+i+j}$ and $v_{t+i+j}$ respectively. I will require
that these expectations be the mathematical expectations of $p$, $m$, and $v$ at $t+j$ conditional
on information available at $t$. Then, aggregating the LHS of (23) over firms also gives
the mathematical expectation of $p_{t+k}$ conditional on information available at $t$;

$$p_{t+k} = \alpha p_{t+k-1} + \frac{1}{\beta \rho c} \sum_{j}(\frac{1}{\beta})^j \{p_{t+k+j} + S + D(m_{t+k+j} - p_{t+k+j} - v_{t+k+j})\}$$

(26)

where

$$S = \frac{\sum_{i} h_i s_i}{\sum_{i} h_i}, \quad D = \left(\frac{\sum_{i} h_i d_i}{\sum_{i} h_i}\right) \sum_{i} h_i$$

$$s_i = (\lambda_i + \mu_i + \alpha_i)/(1 + \beta_i).$$

A rational expectations equilibrium is, in this model a sequence of expected price
levels that enter the RHS of equations (26), and which are equal to the LHS of the
corresponding equation (26). Hence, if these expectations are held, they will be the
mathematical expectations of the price levels. It must also be noted that
if the consistency of the expectations was not imposed, almost any price level could be
the equilibrium price level at time $t$. That is, if one is allowed to pick at will the values
of $(m_{t+k}, v_{t+k})$ then one can also pick almost at will a $p_t$ that satisfies equation (26).

The rational expectations equilibrium can now be computed as the solution to
equation (26). It is a difference equation for the expected price level that is driven by
the expected levels of money balances and of the desire to hold real money balances.

One can rewrite equation (26) as:

$$\left[\frac{1}{\beta \rho c} \frac{1}{1 - 1/\beta} - (1 - \alpha)\right]p_{t+k} = \frac{D}{\beta \rho c} \frac{1}{1 - 1/\beta L}, (m_{t+k} - v_{t+k} + S/D)$$

(27)

Multiplying both sides of (27) by $(\beta L - 1)$:

$$\left[1 - \left(\frac{D}{\rho c} + \alpha + \beta L + \alpha \beta L^2\right)\right]p_{t+k} = -\frac{LD}{\rho c} [m_{t+k} - v_{t+k} + S/D].$$

Factoring the LHS:

$$(1 - \gamma L)(1 - \delta L) p_{t+k} = -\frac{LD}{\rho c} [m_{t+k} - v_{t+k} + S/D]$$

(28)

where

$$\delta + \gamma = \alpha + \beta + \frac{D - 1}{\rho c} = 1/\rho + 1 + D/\rho c$$

$$\delta \gamma = \alpha \beta = 1/\rho$$

and therefore:

$$(1 - \delta)(1 - \gamma) = -D/\rho c.$$  (29)

Once again the two roots are positive; one is larger and the other smaller than one.
A similar diagram to Figure 1 can be used to study the change in the roots as the parameters $D$, $c$, and $\rho$ change. Let the smaller root be $\gamma$. As either $c$ decreases or $D$
increases, $\gamma$ goes down while $\delta$ goes up. As before, to satisfy the transversality condition
(19), I solve forward with the unstable root which is equivalent to dividing both sides
of equation (29) by \((1 - \delta L)\). This yields:

\[ p_{t+k} = \gamma p_{t+k-1} + \frac{D}{\delta p_c} \sum_{j=0}^{\infty} \left( \frac{1}{\delta} \right)^j (m_{t+k+j} - v_{t+k+j} + S/D) \]  

(30)

As can be seen from the definition of \(S\) and \(D\) together with (12), \(m_{t+k} - v_{t+k} + S/D\) is the price level that would be expected at time \(t\) to prevail at time \(t+k\) if there were no costs to changing prices. So, the price level at \(t+k\) is expected to be a function of the previous price level and of the ulterior "desired" price levels. In other words, the price level is expected to slowly adjust towards the price levels that would prevail in the absence of costs to changing prices.

Note that \(\gamma\) is not necessarily equal to \(\alpha\). This is so because the price level at \(t\) appears both on the RHS of equation (26) and on the LHS as long as \(D\) is different from one. Therefore the amount of the price level of the previous period contained in today's price level is a function of how strongly "excess" real money balances affect the demand for goods. Given \(\alpha\), the higher is \(D\), the higher is the effect of wealth on demand, the lower is \(\gamma\), and therefore the faster is the adjustment to the "desired" price level.

Remembering that the price expected at \(t\) for \(t\) is the actual price level at \(t\), the path for \(p_t\) is:

\[ p_t = \gamma p_{t-1} + \frac{D}{\delta p_c} \sum_{j=0}^{\infty} \left( \frac{1}{\delta} \right)^j (m_{t+j} - v_{t+j} + S/D). \]  

(31)

The rest of this paper is devoted to the analysis of (31) along with the path for output that it implies.

5. COMPARATIVE DYNAMICS

In this section I will show that changes in money and in \(v\), whether anticipated or not, affect real output. I will consider two cases. First, I will study a simple stochastic process for \((m-v)\), then I will analyse the reaction of prices and output to the announcement that money will increase sometime in the future.

Case 1 ((\(m-v\)) follows a random walk with drift).

Let \((m_t - v_t)\) follow the process given by:

\[ m_t - v_t = \theta + (m_{t-1} - v_{t-1}) + \epsilon_t \]  

(32)

where \(\{\epsilon_t\}\) is a sequence of independently identically distributed normal variates with mean zero while \(\theta\) is a constant. I will show that increases in both \(\epsilon_t\) and \(\theta\) raise equilibrium output. Throughout the analysis, it will be assumed that the \(\epsilon_t\)'s and \(\theta\) are small enough so that the resulting level of output is feasible. If the \(\epsilon_t\)'s or \(\theta\) were too big the approximations that lead to (15) would cease to be valid and firms which use rule (31) would be forced to ration their customers. Instead, when the \(\epsilon_t\)'s and \(\theta\) are small levels of output above the level which constitutes an equilibrium in the absence of costs of changing prices will be feasible as shown in Section 3. When \((m-v)\) is given by (32) the expectation at \(t\) of \(m_{t+j} - v_{t+j}\) is given by:

\[ m_{t+j} - v_{t+j} = \theta (f+1) + (m_{t-1} + v_{t-1} + \epsilon_t). \]  

(33)

Substituting (33) into (30) and using (29) one obtains the expectation of \(p_{t+k}\) at \(t\):

\[ p_{t+k} = \gamma p_{t+k-1} + (1 - \gamma) \left[ m_{t-1} - v_{t-1} + S/D + \epsilon_t + \theta (k+1) + \frac{\theta}{(\delta - 1)} \right]. \]  

(34)
The solution to the non-homogeneous difference equation (34) is:

\[ p_{t+k} = \gamma^{k+1} p_{t-1} + (1 - \gamma^{k+1}) \times \left( m_{t-1} - u_{t-1} + S/D + \varepsilon_t + \frac{\theta}{\delta - 1} - \frac{\gamma \theta}{1 - \gamma} \right) + \theta(k + 1). \]  

On the other hand, using (14), (33) and (34) the expectation of \( q_{t+k} \) held at \( t \) is:

\[ q_{t+k} = a - \sum_i d_i \left[ \frac{(1 - \delta \gamma)}{(\delta - 1)(1 - \gamma)} \right] \]

where \( a = \sum d_i \).

Output in the absence of costs of changing prices \( q^* \) is given by \( a - \sum_i d_i (S/D) \) which can be verified by substituting (26) into (12) and (14). Therefore the difference between output expected at \( t \) and \( q^* \) is:

\[ q_{t+k} - q^* = \left( \sum_i d_i \right) \left[ \gamma^{k+1} (m_{t-1} - u_{t-1} - p_{t-1} + \varepsilon_t + \frac{(1 - \gamma^{k+1}) \theta (1 - \delta \gamma)}{(\delta - 1)(1 - \gamma)} \right]. \]

The first term on the RHS of (27) captures the effect of the innovation \( \varepsilon_t \) on the path of expected outputs while the second embodies the effect of \( \theta \). I discuss these in turn.

In the absence of costs of changing prices, an increase in \( \varepsilon_t \) translates one for one into an increase in \( p_t \). As firms spread their costs of price changes, an increase in \( \varepsilon_t \) leads to small increases in prices from \( t \) on. Therefore, \( (m_t - p_t - u_t) \) is highest at \( t \) and then declines. So output rises with \( \varepsilon_t \) and then slowly returns to \( q^* \). Uncorrelated disturbances generate correlated output movements. The model, like the one in Lucas (1975) produces an "endogenous" business cycle. The \( \varepsilon_t \)'s also generate fluctuations in relative prices. By (5) and (14), those firms whose \( d_i \) is large adjust their prices mostly towards the present and future levels of money balances, while those whose \( d_i \) is small use mainly the price level as their target. Therefore, at \( t \), those firms with a large \( d_i \) respond more to \( \varepsilon_t \) than the others.

It is worth noting that when \( \theta = 0 \), (39) implies that \( p_t = \gamma p_{t-1} + (1 - \gamma) p^*_t \), where \( p^*_t \) is the price level "desired" at \( t \), namely: \( (m_{t-1} - u_{t-1} + S/D + \varepsilon_t) \). This equation is exactly the one used in the MPS model according to DeMenil and Enzler (1972). This equation has come under attack by McCallum (1979) for being ad hoc. While the preceding discussion makes this criticism less valid it must be recognized that the U.S. money supply cannot be described by a random walk.

Since \( (\delta - 1)(1 - \gamma) \) is positive and \( \delta \gamma \) is equal to \( 1/\rho \), unless firms do not discount the future \( (\rho = 1) \), an increase in \( \theta \) raises the steady state level of output. This can be explained as follows. By letting their prices grow at the rate \( \theta \), the monopolists guarantee themselves constant profits from operations (revenues from sales minus costs of production). By letting prices grow by slightly less than \( \theta \) in the early stages the firms decrease their costs due to price changes while lowering profits from operations. This loss in profits from sales will continue until infinity given that it will never be optimal to increase prices by more than \( \theta \) in any given period. If the discount factor is less than \( 1 \), the discounted stream of losses from sales that results from increasing prices by less than \( \theta \) in the early stages, is smaller than the stream of benefits that accrue from smaller changes in prices during the period that gets counted most heavily in the present value calculation. If, instead, \( \rho \) is one, any change in the level of real profits from operations which will be incurred forever will have an infinite present value and it will never be advantageous for the monopolist to increase its prices by less than the rate of monetary expansion. Therefore, prices go up by \( \theta \) even in the first period.
Case 2 (The effect of an anticipated change in the future money stock)

Suppose that the money stock and \( v \) are known with certainty to remain constant until time \( T \). From \( T \) on the money stock will be equal to the money stock at time zero multiplied by \( e^{\mu t} \) where \( \mu \) is taken to be positive for expositional purposes. This information is revealed at time zero. Suppose, for simplicity, that at time minus one the price level was such that output was at \( q^* \). Then, using equation (31) the aggregate price levels will be:

\[
p_t = m_0 - v_0 + S/D + (1 - \gamma)(1/\delta)^{T-t} \frac{1 - (\gamma/\delta)^{t+1}}{1 - (\gamma/\delta)} \mu, \quad t = 0, 1, 2, \ldots, T. \tag{38}
\]

Equation (38) makes it clear that the price level starts rising at time 0 even though the expansion in the money supply is due at time \( T \). Therefore the announcement of the expansionary policy is contractionary, if believed, since it induces a decline in real money balances. Until the time the expansion in money actually occurs output will therefore be below \( q^* \).

Instead, at time \( T \), output will actually be above \( q^* \) as can be seen from the following argument. Output at \( T \) would be equal to \( q^* \) if the coefficient of \( \mu \) in (38) when \( t \) is equal to \( T \) is unity. Instead, this coefficient is less than \((1 - \gamma)/(1 - \gamma/\delta) \) which in turn is less than one since \( \delta \) is bigger than one. Therefore prices are lower than those that lead to \( q^* \) and output is higher than \( q^* \). The monopolists try to spread the losses from inevitable price changes over time. In equilibrium some of these costs are incurred after \( T \).

6. THE LABOUR MARKET

So far, only good were used as inputs into the production of goods. This assumption was made mainly for simplicity. In this section it is shown that the results of the paper carry over to an economy in which labour is the only factor of production and in which the labour market is competitive.

Let the production function of firm \( i \) be:

\[
Q_{it} = N_i^{1/2}, \quad i = 1, \ldots, n \tag{39}
\]

where \( N_i \) is the amount of labour hired by firm \( i \) at \( t \). Then the demand for aggregate labour is given by:

\[
N_t = \sum_i Q_{it} = \left( \frac{M_i}{P_t V_t} \right)^{2d} \sum_i A_{it}^2 \left( \frac{P_i}{P_t} \right)^{-2b_i} = \left( \frac{M_t}{P_t V_t} \right)^{2d} G_1. \tag{40}
\]

Where the first equality is obtained by using (1) and the assumption, which will be made in this section, that \( d_i \) is equal to \( d \) for all firms. The last term is obtained by approximating a weighed sum of relative prices by the time invariant constant \( G_1 \).

Let the supply of labour be given by:

\[
N_t = G_2 \left( \frac{W_t}{P_t} \right)^f \tag{41}
\]

where \( W_t \) is the wage at \( t \) while \( G_2 \) and \( f \) are parameters. Then if the labour market is competitive, real wages will be given by:

\[
\frac{W_t}{P_t} = G \left( \frac{M_t}{P_t V_t} \right)^{2d/f} \quad G = \left( \frac{G_1}{G_2} \right)^{2d/f} \tag{42}
\]

In the absence of costs of changing prices, firm \( t \), taking the wage and \( M_t/P_t V_t \) as given,
would set its price equal to $\bar{P}_t$:

$$\bar{P}_t = 2\Lambda_t WtQ_t = (2\Lambda_t GA_t)^{1/(1+b_t)}P_t\left(\frac{M_t}{P_tV_t}\right)^{d(f+2)/f(1+b_t)} \tag{43}$$

where the second equality follows from (42). Using (42), real profits from operations can be written as:

$$\pi\left(\frac{P_t}{P_t^d}, \frac{M_t}{P_tV_t}\right) = \frac{P_t}{P_t^d} A_t\left(\frac{P_t}{P_t^d}\right)^{-b_t} A_t\left(\frac{P_t}{P_t^d}\right)^{-b_t} \left(\frac{M_t}{P_tV_t}\right)^d$$

$$-G\left(\frac{M_t}{P_tV_t}\right)^{2d/f} \left[ A_t\left(\frac{P_t}{P_t^d}\right)^{-b_t} \left(\frac{M_t}{P_tV_t}\right)^d \right]^2$$

$$= \pi\left(\frac{P_t}{P_t^d}, \frac{M_t}{P_tV_t}\right) \frac{(b_t + 1)(b_t - 1)}{2\Lambda_t G} (2\Lambda_t GA_t)^{2/(1+b_t)} Z^{2d(f+1-b_t)/f(1+b_t)}(P_t - \bar{P}_t)^2 \tag{44}$$

where $Z$ is the level of $(M_t/P_tV_t)$ which would prevail in the equilibrium without costs of changing prices. The approximations that lead to (44) are identical to those that lead to (15). Moreover, (44) says that in an economy in which labour is the only factor of production, the firm’s optimization problem in the presence of costs of changing prices is essentially identical to the problem presented in Section 6. Hence, the equilibrium can be derived in the same fashion.

Again, increases in $m$ are expansionary because they raise $\bar{P}_n$ by more than they raise $P_t$. They raise output by raising $M_t/P_tV_t$. This leads to higher real wages since more labour must be employed at higher levels of output. Therefore, the real wage will be positively related to output.

7. CONCLUSIONS

This paper has presented a model of an economy that is characterized by fluctuations in aggregate output as responses to fully perceived “nominal” shocks.

Both the textbook “Keynesian” model of Branson (1979) which relies on non-rational expectations and the Lucas model are based on a varying supply curve of labour over the business cycle. At certain times, those with relatively high prices, workers misperceive their current return to working to be higher than usual. In these periods they work at lower real wages and they work more, thereby increasing output. Demand shocks only affect GNP by first confusing producers about their trading opportunities. It is somewhat implausible that these misperceptions can affect output by as much and for as long as is necessary to explain actual business cycles.

Instead, models in which producers are aware of their true trading opportunities, at least insofar as these are affected by the value of aggregate statistics concerning the present, seem more desirable. Here, this information is made available to producers. Instead, it is the perception on the part of producers that changing prices is costly which leads to business cycles in response to nominal shocks. These perceptions may well be reasonable if customers react unfavourably to such prices changes.

This model may be extended in various directions:

The goods could be assumed durable and the monopolists, if not the households, allowed to keep inventories. They would choose their level of inventories optimally taking into account both the convexity of their cost function and the cost to changing prices. The resulting equilibrium would be a joint stochastic process for output, the price level and the aggregate level of inventories. This joint stochastic process would be driven by monetary shocks as in this paper.
The firms could be subjected to firm-specific shocks. If, in addition, they were unable to observe the aggregate statistics, their inference problem when choosing prices would become similar to the inference problem faced by a Phelpsonian islander. This would probably yield an even slower response of the price level to unsystematic nominal shocks.

Producers could face lower than infinite costs to rationing consumers. Then they might occasionally choose to keep their prices near their previous prices while turning away some consumers. These rationed consumers would, in turn, increase their demands in other markets. Whether there would, in such a model, ever be any rationing in equilibrium is an open question.

First version received February 1981; final version accepted January 1982 (Eds.).

This paper is a revised version of Chapter II of my Ph.D. Dissertation which carries the same title. I am indebted to Alan Blinder, William Branson, Gregory Chow, Ed Green, Oliver Hart, Stephen Keafer, Robert Porter, and two anonymous referees for helpful comments and suggestions.

NOTES

1. Specifically, (1) would be the system of demand equations characterizing consumers if these maximized in each period a utility function which depended on their current consumption and on their end-of-period money balances. (1) might also arise if consumers had finite lifetimes as in the consumption-loans model in which money is just a store of value.

2. This is a standard assumption in models with differentiated products (e.g. Dixit and Stiglitz (1977)).

3. An increase of all $q_i$s by an amount proportional to $d_i$ also leaves output unaffected. Such a change is indistinguishable from a change in the desire to hold real balances.

4. A model similar to this one can be used to study the rigidity that results from the monopolists being forced to set their prices before observing their demand curve. This is the rigidity focused on by Gordon and Hynes (1970).

5. This results from the existence of monopolies in the intermediate goods sector. This result is also present in a model by Hart (1980) in which workers have market power.

6. A conceivable mechanism that translates the unfavourable effect of price changes on customers into a cost to the monopolist who changes his prices is the following. Suppose people like to take time to think between the time they see a price and the moment of actual purchase. Upwards movements in prices will then be followed by a period of low demand in which people are digesting the new information and deciding whether they wish to buy at the new prices. Obviously people will only take time to think about the desirability of purchasing a particular item if they think the probability is low that its price will change. Consumers will therefore make up their minds faster in periods of high inflation. The cost of changing prices should therefore decrease as the rate of inflation increases.

7. The curvature of the function that gives the cost of adjustment has important implications as shown by Rothchild (1971) and Folkerts-Landau (1981). In particular convex costs of adjustment like those used in this paper lead to gradual price changes. Meanwhile, fixed costs per price change (which are concave in the price change) lead to abrupt and irregular changes in individual prices.

8. The monopolists must therefore know the effect of all variables on the stochastic process which describes $p$. A random movement in $θ$ have no effect on output as long as $θ$ follows a random walk. This is so because in the absence of discounting, prices always rise by $θ$ when $(m - v)$ is expected to continue to rise at the rate $θ$. Hence $(m - v − p)$ and $q$ are unaffected by $θ$.

REFERENCES


You have printed the following article:

**Monopolistic Price Adjustment and Aggregate Output**
Julio J. Rotemberg
Stable URL:
[http://links.jstor.org/sici?sici=0034-6527%28198210%2949%3A4%3C517%3AMPAAAO%3E2.0.CO%3B2-9](http://links.jstor.org/sici?sici=0034-6527%28198210%2949%3A4%3C517%3AMPAAAO%3E2.0.CO%3B2-9)

This article references the following linked citations. If you are trying to access articles from an off-campus location, you may be required to first logon via your library web site to access JSTOR. Please visit your library's website or contact a librarian to learn about options for remote access to JSTOR.

**Notes**

2 **Monopolistic Competition and Optimum Product Diversity**
Avinash K. Dixit; Joseph E. Stiglitz
Stable URL:
[http://links.jstor.org/sici?sici=0002-8282%28197706%2967%3A3%3C297%3AMCAOPD%3E2.0.CO%3B2-%23](http://links.jstor.org/sici?sici=0002-8282%28197706%2967%3A3%3C297%3AMCAOPD%3E2.0.CO%3B2-%23)

7 **On the Cost of Adjustment**
Michael Rothschild
Stable URL:
[http://links.jstor.org/sici?sici=0033-5533%28197111%2985%3A4%3C605%3AOTCOA%3E2.0.CO%3B2-H](http://links.jstor.org/sici?sici=0033-5533%28197111%2985%3A4%3C605%3AOTCOA%3E2.0.CO%3B2-H)

**References**

**A Theory of Monopolistic Price Adjustment**
Robert J. Barro
Stable URL:

**NOTE:** The reference numbering from the original has been maintained in this citation list.
Monopolistic Competition and Optimum Product Diversity
Avinash K. Dixit; Joseph E. Stiglitz
Stable URL:
http://links.jstor.org/sici?sici=0002-8282%28197706%2967%3A3%3C297%3AMCAOPD%3E2.0.CO%3B2-%23

The Estimation of Partial Adjustment Models with Rational Expectations
John Kennan
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28197911%2947%3A6%3C1441%3ATEOPAM%3E2.0.CO%3B2-X

An Equilibrium Model of the Business Cycle
Robert E. Lucas, Jr.
Stable URL:
http://links.jstor.org/sici?sici=0022-3808%28197512%2983%3A6%3C1113%3AAEMOTB%3E2.0.CO%3B2-5

Monetarism, Rational Expectations, Oligopolistic Pricing, and the MPS Econometric Model
Bennett T. McCallum
Stable URL:
http://links.jstor.org/sici?sici=0022-3808%28197902%2987%3A1%3C57%3AMREOPA%3E2.0.CO%3B2-X

On the Cost of Adjustment
Michael Rothschild
Stable URL:
http://links.jstor.org/sici?sici=0033-5533%28197111%2985%3A4%3C605%3AOTCOA%3E2.0.CO%3B2-H

Inflation and Costs of Price Adjustment
Eytan Sheshinski; Yoram Weiss
Stable URL:
http://links.jstor.org/sici?sici=0034-6527%28197706%2944%3A2%3C287%3AAICOPA%3E2.0.CO%3B2-%23

**NOTE:** The reference numbering from the original has been maintained in this citation list.