Stochastic Technical Progress, Smooth Trends, and Nearly Distinct Business Cycles

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This paper studies a model of random technical progress where technology diffuses at realistically slow rates. It fits smooth trends to the sum of GDP series generated by this model and series representing transitory, or cyclical, fluctuations. Detrended GDP is then largely unrelated to technical progress. The detrending method proposed by Rotemberg (1999) reconstructs cyclical variations somewhat more accurately than the HP filter. With sufficiently slow diffusion it is also more accurate than a method based on VARs fitted to hours and GDP growth. Consistent with the model’s predictions, permanent shocks initially depress both hours and output in these VARs. (JEL E13, O31, O40)

This paper investigates whether it is possible to entertain simultaneously two attractive views about U.S. GDP. The first is that random technical progress leads the long-run level of GDP to be stochastic. The second is that deviations of GDP from a fitted smooth “trend” are mostly attributable to shocks whose effect is temporary. This means that the shocks that cause long-term growth do not have important effects on detrended GDP.

To show that these two views are not incompatible, I first compute how output responds to technical innovations when these diffuse slowly. Such a slow diffusion has been demonstrated in numerous examples discussed by Everett M. Rogers (1995). I then construct artificial GDP series by adding transitory fluctuations to series that are only affected by technical progress. The key result of the paper is that the detrending of these artificial GDP series yields “cyclical” series that are essentially independent of the shocks that affect the long-term level of GDP.

This conclusion appears different in spirit from one of the important conclusions of the real-business-cycle (RBC) literature. Since Edward C. Prescott (1986), this literature has stressed that persistent changes in technological opportunities can have important cyclical effects. Indeed, Prescott (1986) shows that a stochastic process for technical progress with attractive empirical properties leads to fluctuations around a fitted trend whose variance is about the same as the variance of actual fluctuations around an identically constructed trend. This literature has thus argued that temporary departures of output from trend may mostly be due to the forces that can also lead to long-term growth. The reason my results differ from this is that I consider a stochastic process for technical progress that is quite different from the one considered by Prescott (1986).

Prescott (1986) lets technological opportunities follow a first-order autoregression that is close to a random walk. He chooses this specification because it approximates the time-series properties of the Solow residual. As has been pointed out numerous times, however, the Solow residual is also affected by forces other than technical progress. In particular, short-run changes in aggregate demand or labor supply can affect Solow residuals if, either alone or in combination, there are departures from marginal cost pricing, there are variations in labor effort that lead to mismeasurement of the labor input, or there are increasing returns to scale.

Instead of using properties of the Solow residual to calibrate the process for technical progress, I seek to calibrate it on the basis of information about the diffusion of innovations. The literature on this topic is extensive: Rogers

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(1995, p. 443) says that there are about 2,700 published empirical studies of technological diffusion, most of which are cited in at least one of the four editions of his book. This literature is mainly concerned with understanding what leads some people to adopt an innovation while others do not. The point of departure of this literature, however, is that people do not immediately adopt even those innovations which are unambiguously valuable ex post. As Rogers (1995, p. 7) puts it “Many technologists believe that advantageous innovations will sell themselves ... and that [the] innovation[s] will therefore diffuse rapidly. Seldom is this the case. Most innovations, in fact, diffuse at a disappointingly slow rate.” In a classic example studied by Zvi Griliches (1957) only 50 percent of Iowa’s corn acreage was planted with hybrid corn by 1938 even though hybrids yielded 20 percent more than the earlier open-pollinated varieties and even though hybrid seeds were first released to Iowa farmers in 1928.¹

Technological discoveries that diffuse slowly through the economy can still have short-run consequences. In particular, a discovery that is expected to raise the long-run level of output lowers people’s current marginal utility of wealth even if it has no impact on the current ability to produce output. As stressed by Rodolfo E. Manuelli (2000), this tends to increase both the consumption of goods and the consumption of leisure so that current output falls.²

The size of this effect obviously depends on

¹See Everett M. Rogers (1995, p. 33) for further details. To give a more modern example, Consumer Reports first tested American cars with antilock brakes in 1972 (Consumer Reports, April 1988). However, by 1988, only 2.8 percent of new U.S. cars were equipped with such brakes (Wards’s Automotive Reports, January 30, 1989). The fraction of new U.S. cars with ABS brakes first exceeded 50 percent in the 1994 model year (Ward’s Automotive Reports, December 26, 1994).

²Elhanan Helpman and Manuel Trajtenberg (1998) consider a different mechanism through which technical discoveries are capable of depressing current output. They take total labor supply as exogenous and suppose that discoveries only affect the production process if the number of workers devoted to R&D is increased so that the discovery is implemented. The result is that discoveries lower the output of consumer goods as workers are moved from production to R&D. Whether the value of goods and services measured by GDP would fall as a result of such a shift then depends on intricacies of national income accounting. The stochastic process followed by technical progress. It depends, in particular, on the extent to which technical progress leads to revisions in the expectation of the long-term level of GDP. I set the standard deviation of this revision in expectations equal to 0.011. This equals the standard deviation of the long-run expectation of the value of the trend in U.S. GDP, when this trend is estimated by the method of Rotemberg (1999), and is somewhat larger than other estimates of the size of permanent changes in technology.

The short-term effects of technical progress also depend on the speed at which technology diffuses through the economy. Unfortunately, the speed of diffusion has been quite different for different innovations and there is no consensus estimate of the average speed of this diffusion. For this reason, I consider three speeds that bracket most of the estimates in Edwin Mansfield (1989). In the fastest, half of the innovation’s adopters do so within 5 years, while in the slowest it takes them 15. The slowest of these has some common features with the trend component of GDP obtained using the technique described in Rotemberg (1999). According to the univariate evidence from this trend component, a positive shock leads to a protracted period of ever-increasing GDP growth, with the maximum growth of GDP taking place about 60 quarters after the shock.

Lastly, the short-run effects of long-run progress hinge on one’s model of the economy, including the values of its parameters. I carry out this study in the context of the familiar one-sector growth model that has been studied extensively in the real-business-cycle literature.³ I modify this model slightly so that technical progress can be interpreted as being due to an exogenous process of adoption of new production techniques. When these techniques are adopted at the speeds I discussed above, their effect on short-run changes in GDP turns out to be fairly small. I show, for example, that the correlation of the changes in detrended GDP

³In Rotemberg (2002) I also consider a two-sector model where innovations take the form of improvements in the output of the capital goods sector, as in Jeremy Greenwood et al. (1988). When calibrated so that technical innovations are reflected only slowly in the quality of new capital goods, the results are quite similar to those presented here.
and the changes in GDP induced by technical progress is only about 0.05 when it takes 15 years for half the innovation to diffuse. When it takes 5 years, this correlation is as high as 0.11.

These relatively weak effects suggest that it is worth seeking methods of detrending such that the cyclical component reflects as accurately as possible the effect of transitory disturbances. A more successful detrending procedure in this sense allows one to form a more accurate assessment of the extent to which a particular macroeconomic model can account for temporary fluctuations in output. This leads me to consider not only the Hodrick-Prescott filter but also the detrending procedure proposed by Rotemberg (1999). Relative to the former, the latter lowers the correlation between the detrended series (which is supposed to represent cyclical fluctuations) and the difference between future and past trend growth. This is attractive if one supposes either that the shocks to the trend are orthogonal to those that cause temporary fluctuations or if changes in trend growth are due mainly to shocks that occurred in the past, and are thus unrelated to current transitory fluctuations. This difference between the procedures may explain why, in fact, the detrending method of Rotemberg (1999) yields somewhat more accurate measures of the temporary fluctuations in output. When it takes 15 years for half the capital to incorporate an innovation, the correlation between the detrended series and the transitory fluctuations in output is 0.83 when using the HP filter while it is 0.94 when using the method proposed in Rotemberg (1999).

The paper proceeds as follows. The next section discusses the model, while Section II calibrates its parameters. Section III shows that equilibrium output is fairly smooth when only slowly diffusing technical progress affects output, while Section IV is devoted to univariate detrending of series containing both the effects of technical progress and of transitory disturbances.

Section V considers a multivariate method for detrending these series. It studies VARs which include GDP growth and the level of hours predicted by the model. Using a method for identifying permanent and transitory disturbances that is based on Olivier Blanchard and Danny Quah (1989), Jordi Galí (1999), and Neville Francis and Valerie A. Ramey (2002), one can measure the business cycle as the effect of the transitory disturbances. Compared to the method of Rotemberg (1999), this is more accurate if the diffusion is relatively fast and less accurate if it is sufficiently slow. I also study whether these VARs capture the short-run effects of technical improvements. I show that, consistent with the model, the disturbances that permanently increase output and productivity are estimated to lower output and hours at first. Section VI concludes.

I. The Model

I let the representative consumer at $t$ maximize a utility function of the form:

$$
E_t \sum_{j=0}^{\infty} \beta[\log(C_{t+j}) - V(H_{t+j})]
$$

where $C_t$ represents consumption at $t$, $H_t$ represents hours of work at $t$, and $V$ is an increasing convex function. To simplify, I assume that there is a unit mass of consumers so that $C$ and $H$ represent both the per capita and the aggregate levels of consumption and hours.

The representative consumer receives any profits from firms in lump-sum fashion. Meanwhile, the real wage $W_t$ equals a markup $\mu^L_t$ times the marginal rate of substitution between leisure and consumption so that

$$
\frac{W_t}{C_t} = \mu^L_t V'(H_t).
$$

Labor market distortions can raise $\mu^L_t$ above one, which is the value of $\mu^L$ when labor is competitively supplied.

Each firm has $N_t$ different types of capital at $t$ and the fraction of capital of type $j$ equals $\rho_j$. Each of these types of capital is associated with a different technology parameter $\xi_j$ so that the output of firm $i$, $Y_i$, is given by

$$
Y_{it} = B \sum_j (\xi_j H_{ijt})^a (\rho_j K_{it})^{1-a} - \Phi
$$

where $K_{it}$ is the capital of firm $i$ at $t$, $H_{ijt}$ is the amount of labor firm $i$ devotes to capital of type $j$ at $t$ while $B, a, \rho_j, \Phi$ are parameters. I include the fixed cost $\Phi$ to ensure that firms earn no
profits on average, even when their price is above marginal cost.

I allow for multiple varieties of capital in (3) to provide an interpretation for the slow diffusion of technology. In this interpretation, the fractions of the different types of capital are exogenous, so the model behaves like a standard model of exogenous technical progress once the firm allocates optimally the total amount of labor it hires, $H_i$, over the different types of capital. Its output is then equal to

$$Y_i = B z_i H_i^a K_i^{1-a} - \Phi$$

where $z_i = \left( \sum_j \rho_j z_j^{a(1-a)} \right)^{(1-a)/a}$.

Firms take factor prices as given. Letting $R_i$ be the price in terms of consumption goods that firms must pay to rent one unit of capital for use during $t$, firms set

$$\alpha B z_i \left( \frac{K_i^a}{z_i H_i} \right)^{1-a} = \mu_i^G W_i$$

$$\beta = (1 - \alpha) B \left( \frac{z_i H_i^a}{K_i^a} \right) = \mu_i^G R_i$$

where $\mu_i^G$ is the markup of price over marginal cost.

I simplify the analysis by supposing that purchasers see all the goods produced by the different producers as perfect substitutes. This means that, with $N_i$ symmetric firms, aggregate output $Y_i$ is

$$Y_i = B (z_i H_i)^a K_i^{1-a} - N_i \Phi$$

Symmetry across firms also implies that (5) and (6) hold when the individual values $K_i$ and $H_i$ are replaced by the aggregates $K$ and $H$. If free entry ensures that profits are zero, (4), (5), (6), and (7) imply that the number of firms and aggregate output equal, respectively,

$$N_i = (\mu_i^G - 1) Y_i / \Phi$$

$$Y_i = \frac{B}{\mu_i^G} (z_i H_i)^a K_i^{1-a}.$$ 

Thus aggregate output behaves as if an exogenous factor $z_i$ multiplies the firms’ labor input. Equation (9) obviously holds also under perfect competition even though individual firm output and the number of firms are indeterminate in this case.

The total amount of capital satisfies the standard accumulation equation

$$K_{t+1} = (1 - \delta) K_t + Y_t - C_t.$$ 

There also exist intermediaries that have the capacity to buy capital with funds that they borrow from the representative consumer at $t$. These consumers are promised that, in exchange, they’ll receive at $t+1$ a fraction of the proceeds from both the rental of capital and the selling of the depreciated capital. I allow this fraction to be smaller than one so that this market imperfection is modeled as in Russell Cooper and João Ejarque (2000). Therefore

$$\frac{1}{C_t} = \frac{1}{\mu_i^G} E_t \beta [R_{t+1} + \delta K_t].$$

From these equations, I derive three standard equilibrium conditions in $C_t/z_t$, $K_t/z_t$, and $H_t$. The first of these conditions follows from combining (5) (as it applies to aggregate factors) and (2),

$$\mu_i^G V'(H_i) H_i^{1-a} C_t / z_t = \alpha \left( \frac{K_i^a}{z_i} \right)^{1-a}.$$ 

The second equilibrium condition is obtained by combining (6) when it is applied to aggregate factors and (11)

$$\frac{1}{C_t} = \frac{1}{\mu_i^G} E_t \beta \frac{1}{C_{t+1}} \left[ \frac{(1 - \alpha)B}{\mu_i^G} \left( \frac{z_i H_i^a}{K_i^a} \right) \right] + 1 - \delta].$$

The third and last equilibrium condition is not invariant to whether one is considering the case
with free entry or whether one is temporarily holding the number of firms fixed. With a given number of firms $N$, using (7) in (10),

\[
\frac{K_{t+1} - z_{t+1}}{z_{t+1}} = (1 - \delta) \frac{K_t - C_t}{z_t} + B \frac{H_t^\alpha}{z_t} \left( \frac{K_t}{z_t} \right)^{1-\alpha} - \frac{N \Phi}{z_t}.
\]

If, instead, one considers the case of free entry, (9) implies that

\[
\frac{K_{t+1} - z_{t+1}}{z_{t+1}} = (1 - \delta) \frac{K_t - C_t}{z_t} + \frac{B}{\mu^L \mu^F} \frac{H_t^\alpha}{z_t} \left( \frac{K_t}{z_t} \right)^{1-\alpha}.
\]

These equations differ only in that increases in hours and capital have a larger effect on output (and thus on either consumption or capital accumulation) if the number of firms does not increase in the way that it would with free entry (where the size of firms is constant).

As in Robert G. King et al. (1988), I study the dynamics of this model by linearizing these equations around a steady state with a constant value of $z_t$, $z_r$, and constant markups $\mu^L$, $\mu^G$, and $\mu^F$. For such a constant growth rate of technical opportunities, equations (12), (13), and (15) can be solved for steady-state values of $C_t/z_t$, $K_t/z_t$, and $H_t$. I denote the logarithmic deviations of $C_t$, $K_t$, and $H_t$ from their steady-state by $\bar{C}_t$, $\bar{K}_t$, and $\bar{H}_t$, respectively. When I consider the effects of random technical progress, I let entry be free and markups be constant. Differentiating (12), (13), and (15) gives

\[
\bar{C}_t + (1 - \alpha + \xi) \bar{H}_t = (1 - \alpha) \bar{K}_t + \alpha \bar{\gamma}_t,
\]

\[
\frac{\gamma K}{Y} \left( \bar{K}_t + \bar{\gamma}_t \right) - \frac{(1 - \delta) K}{Y} \bar{K}_t + \frac{C}{Y} \bar{C}_t = (1 - \alpha) \bar{K}_t + \alpha (\bar{H}_t + \bar{\gamma}_t),
\]

\[
\frac{\mu^F \gamma}{\mu^F \gamma - \beta(1 - \delta)} E_t(\bar{C}_{t+1} - \bar{C}_t + \bar{\gamma}_t) = \alpha E_t(\bar{H}_{t+1} - \bar{K}_{t+1} + \bar{\gamma}_{t+1})
\]

where $K/Y$ and $C/Y$ represent the steady-state ratios of capital and consumption to output, respectively, $\gamma$ is the steady-state value of $z_{t+1}/z_t$, and $\xi$ is the inverse Frisch elasticity of labor supply $Y'H'/Y'$.

These three equations can be used to compute the equilibrium paths of $\bar{C}_t$, $\bar{H}_t$, and $\bar{K}_t$ for given expectations about the evolution of $\bar{\gamma}_t$. It is worth noting that these three equations are independent of the average values of $\mu^L$ and $\mu^G$. The latter does not matter because, as can be seen in (9), free entry leads the model to be one of constant returns: a proportional increase in labor and capital is accompanied by a proportional increase in the number of firms so that output rises proportionately as well.

Using (9) to compute $Y_t/Y_{t-1}$ and differentiating, the log difference between output growth at $t$ and its steady-state value, which I denote by $\gamma_t$ is

\[
\gamma_t = (1 - \alpha)(\bar{K}_t - \bar{K}_{t-1} + \bar{\gamma}_t) + (1 - \alpha)(\bar{H}_t - \bar{H}_{t-1} + \bar{\gamma}_t).
\]

I suppose that output is also affected by shocks that only have a temporary effect. I use $\bar{\gamma}_t$ to denote the resulting deviations of output from the level induced by $\gamma$. These deviations could, for example, be the result of changes $\mu^L$, $\mu^G$, and $\mu^F$. In that case, one can obtain the effect of the changes in these wedges for a constant number of firms by differentiating (12), (13), and (14). One can also start with a stochastic process for $\bar{\gamma}_t$.

\[
H(L)\bar{\gamma}_t = \epsilon_t
\]

where $H(L)$ is a polynomial in the lag operator $L$ whose first term is equal to one and use the model to study the combinations of fluctuations in efficiency wedges that can give rise to these fluctuations. Alternatively, one can suppose that fluctuations in $\bar{\gamma}_t$ are due to temporary changes in the productive efficiency of firms. For what follows, the interpretation one gives to these fluctuations is unimportant.

II. Calibration

There are two quite different types of parameters that play a role in the models I simulate.
The first are the behavioral parameters that act as coefficients in the linearizations discussed in the previous section. The second are the parameters that govern the evolution of \( \hat{y}_t \) and \( \bar{y} \). As far as the former are concerned, I set \( \mu_\pi = 1 \) and choose behavioral parameters which are within the range considered in the real-business-cycle literature (see King and Rebelo, 1998). Thus, \( \beta, \xi, \alpha, \delta, \gamma, \) and \( C/Y \) equal 0.99, 1, 0.7, 0.03, 0.01, and 0.7, respectively.

As I discussed in the introduction, I let technology diffuse slowly, as it appears to do in practice. Before calibrating this diffusion process, it is worth reinterpreting the changes in \( z_t \) in (4) as involving the gradual adoption of technical improvements. Each type of capital in (3) can be thought of as incorporating a different mix of innovations and each included innovation raises the capital's \( \bar{z} \). Suppose that each innovation \( m \) is associated with a \( \bar{z}_m > 1 \) and that the \( \bar{z} \) of any particular type of capital equals the product of the \( \bar{z} \)'s of the innovations it includes. Then, at each time \( t \) an innovation characterized by \( \bar{z}_t \) becomes available. From that point on, some types of capital contain this innovation while others do not. Let the fraction \( \rho_{t+\tau} \) of capital at \( t + \tau \) contain this innovation while the rest does not. Suppose further that the fraction of capital that contains any other innovation is independent of whether it contains \( \bar{z}_t \) or not. The effect of \( \bar{z}_t \) on \( z_{t+\tau} \) is then

\[
(1 - \rho_{t+\tau} + \rho_{t+\tau} \bar{z}_t \alpha^a/(1 - \alpha) \alpha^a/\alpha).
\]

Using this interpretation, one can use information about the diffusion of innovations, which leads \( \rho_{t+\tau} \) to rise with \( \tau \), to calibrate the stochastic process for \( z_t \). One advantage of this approach is that, for small values of \( \bar{z}_t \), the expression above is approximately linear in \( \bar{z}_t \) and \( \rho_{t+\tau} \). Thus, in this case, the amount of time it takes half the capital to incorporate the innovation (which can be interpreted as having half the population adopt it) equals the amount of time it takes for \( z_t \) to reach half of its steady-state level after a shock.

While it is known that the diffusion of innovations often has an S shape, so that the speed of diffusion first rises and later falls, it is difficult to know the average speed of diffusion. The source of this difficulty is that innovations are heterogeneous and that the sample of innovations for which there is somewhat consistent data may not be representative. Mansfield (1989) indicates that it took 5 years from the point of introduction for half the potential adopters to start using pallet loading machines. By contrast, it took 15 years for half the potential adopters to use by-product coke ovens. The basic oxygen process for making steel was first used commercially in the United States in 1954 but, by 1968, only about 40 percent of U.S. steel output was produced by this method.

As innovations diffuse, further improvements often take place. If one treats these subsequent improvements as expected at the time of the initial innovation, a simple count of the number of adopters understates the extent to which increases in \( z \) due to the original invention are delayed relative to the date of invention. On the other hand, the extent to which subsequent improvements are forecastable is open to question. Thus, while the method I use to extract information about the evolution of aggregate technology from information about speeds of adoption has a certain appeal, alternative specifications still deserve exploration.

An alternative method for calibrating the process for \( z \) is to use information about "trend" movements in the logarithm of U.S. GDP. A
particular method for decomposing series into the sum of a trend and a cycle is proposed in Rotemberg (1999). This method imposes orthogonality between the cycle at \( t \) and the difference between the value of the trend at \( t \) and the mean of the trend values at \( t + k \) and \( t - k \). It thereby ensures that "temporary" movements in the trend are orthogonal to the measured cycle. The method also seeks to make the trend smooth and to make cycles relatively short. Rather than imposing either of these conditions, the method chooses the cycle \( \tilde{y}_t^c \) to minimize a weighted average of losses due to lack of trend smoothness (i.e., the mean of the squared second difference in the trend) and losses due to having cycles that last a long time (i.e., the covariance of the cycle at \( t \) and the cycle 16 quarters ago). The relative weight on these two losses ensures that the trend and the cycle are orthogonal in the way described above.

In other words, starting with a series \( y_t \), \( \tilde{y}_t^c \) minimizes

\[
\sum \lambda [(y_t - \tilde{y}_t^c) - 2(y_{t-1} - \tilde{y}_{t-1}^c)] + (y_{t-2} - \tilde{y}_{t-2}^c)^2 + (\tilde{y}_t^c - \bar{y}^c)(\tilde{y}_{t-16}^c - \bar{y}^c)
\]

where \( \lambda \) is the smallest value which ensures that

\[
\sum (y_t^c - \bar{y}^c)[2(y_t - y_t^c) - (y_{t-1} - \tilde{y}_{t-1}^c) - (y_{t+k} - \tilde{y}_{t+k}^c)] = 0
\]

and where \( \bar{y}^c \) is the mean of \( y_t^c \) while \( k \) is set to 5 quarters, though other small positive values of \( k \) give similar answers. Rotemberg (1999) argues that this problem has a well-defined solution because (22) is negative if \( \lambda \) is small enough while, if the true trend is not linear, it is positive for \( \lambda \) sufficiently large.

Figure 1 shows the evolution of the resulting change in the trend of the log of U.S. GDP from 1947:1 to 1998:1. From a statistical point of view, this growth rate (or log difference) seems well described by an AR(5) process. In particular, a regression explaining this log difference measured, for example, as in Susanto Basu et al. (1998). The problem is that, if technological disturbances have only very small effects at first, such methods may not measure accurately either the first appearance of an innovation or the length of its effects.

with five lags of this log difference yields the following

\[
\Delta y_t^c = 4.11 \Delta y_{t-1}^c - 6.59 \Delta y_{t-2}^c + 5.1 \Delta y_{t-3}^c - 1.86 \Delta y_{t-4}^c + 0.24 \Delta y_{t-5}^c
\]

\[s.e. = 8.84e - 8\]

where \( \Delta y_t^c \) is the rate of growth of trend GDP at \( t \) and the constant is imprecisely estimated and insignificantly different from zero. Further lags are not statistically significant when added to this regression. While rounding leads the coefficients in (23) to add to 1.0, they do, in fact, add up to something less than this. A more revealing way to describe this stationary process is thus to display the moving average coefficients, which describe how (trend) output growth responds to a unit impulse that takes place in the first period.10

Interestingly, the first difference of the HP trend of GDP also fits an AR(5) fairly well. Its residual is quite a bit larger, however. Its standard error equals 4e-6. Moreover, the moving average representation of this fitted process involves much stronger short-run effects. The main reason I do not use the HP trend is that trend growth of GDP obtained by this method is strongly correlated with the HP cycle. It would thus be unattractive to create artificial GDP series by combining artificial trends that mimic the properties of the HP trend with cyclical series that are independent of these constructed trends.
This response is displayed in Figure 2. This figure shows that the growth rate rises initially and reaches a maximum of 1.915 after 63 quarters. The growth rate then falls back towards its steady-state value, though it overshoots it as the convergence to the steady state involves damped oscillations. The long-run effect of a unit impulse on the level of output is 1.25e5.

While this long-run effect seems huge, it is important to remember that the standard deviation of the residuals in (23) is only 8.84e-8. This means that these residuals induce a standard deviation of the innovation in the level of long-run output that equals (1.25e5)(8.84e-8) or 0.01. This is larger but not much larger than the value of 0.007 that Rotemberg and Woodford (1996) estimated for the standard deviation of the innovation in long-run output using a VAR containing GDP growth, the ratio of consumption to GDP, and linearly detrended hours worked.

Based on these observations, I let the stochastic process for $\gamma_t$ be given by

$$ (1 - \lambda_1 L)(1 - \lambda_2 L)(1 - \lambda_3 L)\gamma_t = \epsilon_t^\gamma $$

(24)

where the $\lambda$'s represent three roots and $\epsilon_t^\gamma$ is an independently and identically distributed (i.i.d.) variable with standard deviation $\sigma^\gamma$. I choose $\sigma^\gamma$ so that the standard deviation of the corresponding innovation in the permanent level of output equals 0.011 while I let $\lambda_1$ and $\lambda_2$ be complex conjugates. In polar coordinates, I set the modulus of these roots close to one and let their angle be small. This ensures that the rate of growth of $z_t$ rises for some time before it starts falling. The higher the modulus, the longer the span over which the rate of growth rises, and thus the longer it takes before a given shock has 50 percent of its effect on the level of $z$. The higher the angle, the higher the frequency of the fluctuations in the growth of $z$ as this rate of growth converges towards zero. This means that, by setting this angle to the small value of 0.013, I ensure that the absolute value of the rate of growth of $z$ is negligible after this rate of growth first equals zero. I also set $\lambda_3 = 0.5$. A higher value of $\lambda_3$ lowers the instantaneous rate of growth of $z$ induced by a positive $\epsilon_t^\gamma$ relative to the peak rate of growth induced by this shock. While this parameter only has a trivial effect on the results, setting it to a positive number ensures that my process mimics a feature of many actual diffusions.

Lastly, I consider three different values for the modulus of $\lambda_1$ and $\lambda_2$. These are 0.915, 0.961, and 0.977 and correspond, respectively, to having 20, 40, or 60 quarters elapse before $\epsilon_t^\gamma$ has half of its effect on the level of $z$. I choose these three values because they bracket most of Mansfield's (1989) estimates for the amount of times it takes for half the potential adopters to adopt an innovation. Using the labels $m^\gamma_{20},$ $m^\gamma_{40},$ and $m^\gamma_{60},$ the speed of diffusions may well depend on economic conditions in general. I neglect this in my analysis both because I am not aware of any evidence bearing on this issue and because this effect ought to be negligible for sufficiently small fluctuations around the steady state. Incorporating this effect would require a departure from the first-order approximation approach I spelled out above.

It might be seen as more desirable to use parameter estimates from the estimation exercises in the diffusion literature. Many of these, like Griliches (1957), fit logistic curves to diffusion data. Unfortunately, these curves implicitly assume that the product or process starts diffusing at minus infinity. This leads authors like Griliches (1957) to estimate parameters by using a sample that includes only observations with a strictly positive level of diffusion. so initial observations are neglected. The diffusion literature also focuses on "diffusion constants," which are the amount of time that elapses between the moment where a product has diffused to 10 percent of its ultimate users and the moment where this diffusion is 90 percent complete. Arnulf Grübler (1991) reports that, for 265 innovations, the mean diffusion constant is 41 years though 40 percent of the products had constants below 30 years. For comparison, the diffusion constants for the three cases I consider are 9.25, 18.5, and 26 years, respectively. Relative to this study, the diffusion speeds I consider are thus relatively rapid.
$m_{40}$ and $m_{60}$ for these three moduli, Figure 3 displays the response of $z$ to $e_z$ for all three processes. The values of $e_z$ are chosen so that, in all three cases, their eventual effect on the level of $z$ equals one. One advantage of the largest of these moduli is that it delays the peak response of output growth to an $e^z$ shock by 47 quarters, which is comparable to that implied by (23). To actually match this peak response time, the modulus of $\lambda_1$ and $\lambda_2$ must be made somewhat larger.\footnote{To match the length of time it takes until the peak response is achieved, it is probably not necessary to use a process such that it takes longer than 60 quarters for $z$ to reach 50 percent of its steady-state level. Rather, what is needed is a stochastic process whose peak rate of growth is delayed further relative to the point where the level reaches half of its steady-state value. This appears to require a higher-order process than the one I have considered.}

I combine the stochastic trend due to $e^z$ shocks with cyclical fluctuations in output. The temporary movements of output I consider are those that emerge from the trend-cycle decomposition defined by (21). Using quarterly data from 1947:1 to 1998:1, and ignoring the constant, detrended output $\hat{z}^t$ appears to be well-described by the following autoregression:

\begin{equation}
\hat{z}^b_t = 0.89 \hat{z}^b_{t-1} + 0.34 (\hat{y}^b_{t-1} - \hat{y}^b_{t-2}) \\
+ 0.15 (\hat{y}^b_{t-2} - \hat{y}^b_{t-3}) + e^b_t \\
R^2 = 0.89.
\end{equation}

Subsequent lags are statistically insignificant. Once again, an appealing way to describe this involves displaying the way $\hat{y}^b$ responds to a unit impulse. This is shown in Figure 4. After increasing by one unit, such a shock leads to further small increases in output in the next two quarters. After this, cyclical output returns fairly rapidly to its mean value. Indeed, after 15 quarters the effect of the shock is almost entirely dissipated.

I assume in my analysis that $e^z_t$ is independent of $\hat{e}^z_t$. This assumption should probably be relaxed. One reason for doing so is that Jacob Schmookler (1966) interprets his evidence as suggesting that positive values of $\hat{e}^b_t$ lead to subsequent increases in patenting, which might correspond to positive values of $e^z_t$.\footnote{Schmookler (1966) reaches this conclusion on the basis of observing that, in time-averaged data, high levels of patenting activity tend to be preceded by high levels of output in patent-using sectors. Using an error-correction model, Paul A. Geroski and C. F. Walters (1995) also find a positive, though statistically insignificant, effect of lagged aggregate industrial production on aggregate patenting activity. They also suggest that, in their annual sample of U.K. patents for the period 1948–1983, these variables are more strongly positively correlated at low frequencies. By contrast, they report that the correlation between the change in industrial production and the contemporaneous change in patents granted is negative and equal to $-0.17$.}

III. The Effect of Variations in $\hat{y}$

I now discuss the responses implied by (16), (17), and (18) to the shocks $e^z_t$. Figure 5 shows
the reaction of the levels of output and hours to changes in $\varepsilon^*_t$ whose size is normalized so that the long-run level of output rises by one unit. Consistent with the response of $z$ itself, the response is more drawn out the higher the modulus of the complex roots.

Figure 5 also shows that an increase in $z$ initially lowers output in most of these specifications. While the size of output declines is largest on impact, output continues to decline for a while longer before it starts rising. The mechanism for this finding is easy to understand, and is quite similar to that in Manuelli (2000). Future increases in technical progress lower the current marginal utility of wealth. They thus lead the representative agent to increase his current consumption as well as his current leisure so that output falls. The rise in consumption is large enough that the capital stock falls as well and this further lowers output.

I now study the smoothness of growth rates of GDP induced by random draws of $\varepsilon^*$ when $\dot{y}$ follows the process in (24). The effect of the history of $\varepsilon^*$ shocks on the log difference of output at $t$ can be written as

$$\Delta y_t = \Theta^y(L) \varepsilon^*_t$$

where the coefficients of $\Theta^y(L)$ depend on the specification. I use the coefficients underlying Figure 5 while ignoring the negligible coefficients on powers of $L$ greater than 500. I construct draws of output growth of length 205 (which corresponds roughly to the length of available quarterly GDP series in the United States) by using a random number generator to obtain realizations of $\varepsilon^*_t$. I then add a linear trend with a coefficient that ensures that the rate of growth of this constructed series equals 0.0079, which is the rate of growth of $y_t^{0.15}$. These constructed series for trend growth are denoted by $\Delta y^\tau$.

I construct 500 series of $\Delta y^\tau$ for each specification. For each of the resulting $\Delta y^\tau$ series, I compute a simple measure of smoothness. This measure is simply the mean of $(\Delta y^\tau_t - \Delta y^\tau_{t-1})^2$. If $y^\tau$ were a straight line, this square of the first difference of $\Delta y^\tau$ would be zero. Averaging across all realizations, this mean equals $113.1e-8, 23.6e-8, 7.5e-8$ for the cases of $m_{20}, m_{40}$, and $m_{60}$, respectively. The reason trends with higher moduli are smoother is that they spread out the increase in $z$ from any increase in $\varepsilon^*$ over a longer time span. For purposes of comparison, the mean of the squared second difference of U.S. GDP from 1947:1 to 1998:1 is $1.47e-4$. Thus, all constructed series are substantially smoother than actual GDP.

Because these trends are so smooth, they tend to have only relatively small movements at cyclical frequencies. One way of seeing this is by applying the HP filter to $y^\tau_t$ series, which are obtained by adding the $\Delta y^\tau_t$ realizations to an arbitrary initial condition. Averaging across realizations, the standard deviations of these HP detrended series equal 0.22 percent, 0.08 percent, and 0.04 percent for $m_{20}, m_{40}$, and $m_{60}$, respectively. These are obviously significantly lower than the standard deviation of the HP filtered U.S. GDP series, which equals about 1.8 percent.

IV. Univariate Analysis of Series Incorporating Technical Progress and Temporary Fluctuations

The series I analyze in this section are the sum of series affected by $\varepsilon^*$ and series that have the same stochastic process as (25). The latter, which I denote by $y^\tau_t$, can be written as

$$y^\tau_t = \theta^\tau(L) \varepsilon^*_t$$

15 As discussed in Rotemberg (1999), these added linear trends end up fully incorporated into the estimated trends. Thus, the only function of these added trends is to make the low-frequency behavior of the constructed $\Delta y$ series similar to that of $y^\tau$. 

where \( \varepsilon \) is an i.i.d. random variable with the same variance as \( \varepsilon^p \), namely, 9e-5 and \( \delta'(L) \) follows from inverting (25). I ignore the coefficients of the powers of \( L \) greater than 50 because they are minuscule. The series I analyze in this section are thus given by

\[
y^m_t = y^c_t + y^f_t
\]

where each of the 500 histories of \( y^m \) that I study is obtained by creating sequences of \( \varepsilon^c \) and \( \varepsilon^f \) using a random number generator.

Before analyzing these series, it is worth discussing briefly the connection between the properties of \( y^m \) and those of the logarithm of U.S. GDP. The two series obviously do not have identical statistical properties, since the initial reaction to \( z \) shocks in the equilibrium models is generally quite different from the initial reaction of GDP to a shock to equation (23). For purposes of comparison, Figure 6 displays the log of the periodogram of the log of U.S. GDP from 1947:1 to 1998:1. I also computed the periodogram for 500 realizations of \( y^m \) for the model with modulus \( m_{60} \). The smooth curve in the center of the displayed lines of Figure 6 contains the log of the mean values (frequency by frequency) of these power spectra. The two lines at the top and bottom are the log of the maximum and minimum realized values of power at each frequency from these 500 observations. Because these lines are quite close to one another, one can conclude that the realized histories of \( y^c \) are broadly similar to the history of U.S. GDP. This is also true for the other moduli I have considered.

I consider two univariate methods for decomposing the histories of \( y^m \) into cyclical components and components that emphasize lower frequencies. The first is the Hodrick-Prescott filter and the second is the decomposition in (21). Both filters recover relatively smooth trends, which suggests that they might be appropriate for reconstructing \( y^c \). Table 1 gives some indicators of the extent to which the cycles and the trends obtained by these methods vary with \( y^c \) and \( y^m \) for the cases of \( m_{20} \) and \( m_{60} \). Except where I note so explicitly, these indicators are obtained by ignoring the first and last 16 observations. As can be seen from the difference in mean squared errors when the boundaries are included, both methods find it more difficult to estimate smooth trends near the boundaries of the sample.

A useful benchmark with which to compare the MSEs is the variance of the change in log GDP, which is the MSE one would obtain for the change in GDP if one estimated this change by a constant. The variance in the change of U.S. GDP and that of the change in \( y^m \) are considerably larger, since they equal about 1.2e-3. On the other hand, even the lowest MSEs in the table are not negligible relative to the variance of the cycle. The cycle variance from (25) is 8.08e-4 so that, ignoring the boundaries, the MSE of the trend from (21) equals about 15 percent of the variance of the signal \( y^c \) in the more favorable \( m_{60} \) case. However, the MSE is not ideal for understanding whether detrending recovers a useful measure of the cycle, because it is affected by the mean error. This mean error can be particularly important when the boundaries are ignored because the mean of these detrended series can then be nonzero. I thus consider other properties of the error in measuring \( y^c \) (or \( y^m \)) which are particularly relevant if one wants to use detrended series to study the properties of business cycles.

To study these properties, I denote by \( \hat{y}^c \) the

16 A majority in the entries for the case of \( m_{60} \) are within 2 percent of the average of the results in these two tables. However, the mean square errors obtained using (21) are between 5 and 20 percent larger than the average of the mean square errors in these tables.
estimated cycle, i.e., the difference between $y^m$ and its trend.

Perhaps the most striking statistics in Table 1 are those that give the $R^2$ in the regression of the change $y^c$ on the change in $y^c$. These $R^2$s imply that the correlations between the change in these series is at most 0.11 in the $m_{20}$ case and at most 0.05 in the $m_{60}$ one. This result is not all that surprising once one knows that the detrended values of $y^c$ have very low variability, since this implies that the detrended movements in $y^m$ must be dominated by movements in $y^c$. Nonetheless, these low correlations have an important implication. If the correlation were high so that the shocks that lead to long-term growth had important effects at business-cycle frequencies, the detrending of time series would mainly be a matter of convenience. Since both the measured trend and the detrended series would be affected by the same forces, one would not be able to give a full account of the effect of these forces by studying either of these components in isolation.

By contrast, if the shocks that lead to long-term growth have relatively little effect on detrended series, then it is possible to hope that one can find a method of detrending that gives relatively accurate measures of $y^c$. The more accurate this measure, the more certain one can be in one’s assessment of the extent to which a particular macroeconomic model accounts for transitory fluctuations. The accuracy of any detrending method, in turn, clearly depends on the process generating $y^c$ and $y^z$. However, insofar as certain techniques yield consistently more accurate measures of $y^c$ for a relatively broad range of stochastic processes, they ought to be preferred. One potential advantage of (21) over the HP filter is that, as shown by King and Rebelo (1993), the HP cycle of many macroeconomic variables such as GDP is highly correlated with temporary movements in the HP trend, i.e., with the difference between the current value of the trend and the average of the trend $k$ quarters ago and $k$ quarters hence. By contrast, (21) and (22) make these uncorrelated for a particular $k$ and yield low correlations for nearby values of $k$. This seems particularly attractive if $y^c$ and $y^z$ are independent.

Whether for this reason or not, (21) does yield somewhat more accurate measures of $y^c$, particularly in the $m_{60}$ case. This is true whether one looks at MSEs or at the more revealing and quite substantial correlations between the $y^c$ and $y^z$. Perhaps the most interesting result concerning this accuracy is the coefficient in the regression of $y^c$ on $y^z$. Using (21), this is very close to 1, which suggests that the best estimate of $y^c$ based on the observation of $y^z$ is $y^z$ itself.

A perhaps less surprising result is that the correlation of $y^c$ and $y^z$ is enhanced by first differencing the two series. This presumably occurs because differencing emphasizes higher frequencies and these are less affected by $y^z$.

<table>
<thead>
<tr>
<th></th>
<th>HP filter</th>
<th>Rotemberg (1999) filter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m_{20}$</td>
<td>$m_{60}$</td>
</tr>
<tr>
<td>MSE of fitted trend</td>
<td>3.0e-4</td>
<td>3.0e-4</td>
</tr>
<tr>
<td>MSE of fitted trend (full sample)</td>
<td>3.2e-4</td>
<td>3.1e-4</td>
</tr>
<tr>
<td>Correlation of actual and fitted cycle</td>
<td>0.827</td>
<td>0.834</td>
</tr>
<tr>
<td>$R^2$ in regression of fitted on actual cycle</td>
<td>0.688</td>
<td>0.699</td>
</tr>
<tr>
<td>Coefficient in regression of fitted on actual cycle</td>
<td>0.582</td>
<td>0.583</td>
</tr>
<tr>
<td>Coefficient in regression of actual on fitted cycle</td>
<td>1.189</td>
<td>1.204</td>
</tr>
<tr>
<td>$R^2$ in regression of $\Delta y^c$ on $\Delta y^z$</td>
<td>0.974</td>
<td>0.980</td>
</tr>
<tr>
<td>Coefficient in regression of $\Delta y^c$ on $\Delta y^z$</td>
<td>0.955</td>
<td>0.954</td>
</tr>
<tr>
<td>$R^2$ in regression of $\Delta y^c$ on $\Delta y^z$</td>
<td>0.006</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Properties of fitted trend growth

|                       | $m_{20}$  | $m_{60}$                | $m_{20}$  | $m_{60}$                |
|                       | 0.324     | 0.165                   | 0.352     | 0.516                   |
| $R^2$ in regression of $\Delta y^c$ on $\Delta y^z$ | 0.651     | 0.843                   | 0.343     | 0.582                   |
| Coefficient in regression of $\Delta y^c$ on $\Delta y^z$ | 0.454     | 0.156                   | 6.713     | 1.927                   |
The result is that the corresponding $R^2$s are above 0.99 when using (21) while the coefficients in this regression are estimated to be essentially equal to 1. Thus, the changes in $y^c$ are essentially all due to changes in $y^c$. This tight link of the growth in the actual cycle and the change in the detrended series corresponds to the low correlation between the change in the detrended series and the change in $y^c$.

I now focus on the extent to which changes in fitted trends, which I denote by $\Delta y^c$, reflect the changes in output induced by technical progress $\Delta y$. The $R^2$s of these regressions are substantially lower than those connecting fitted and actual cycles. As one goes from $m_{20}$ to $m_{60}$, this $R^2$ rises when (21) is used, while it falls in the HP case. This may be because, in the latter, the trend picks up a larger fraction of the movements in the cycle $y^c$. Since the fluctuations in $\Delta y$ are smaller in the $m_{60}$ case, the fraction of the HP trend changes due to changes in $\Delta y^c$ is smaller. The method based on (21), by contrast, captures the trend more accurately when this trend is smoother. Even here, however, the coefficient one obtains when running a regression of actual trend growth on fitted trend growth is quite different from one, suggesting that it may be difficult to use these fitted trends to obtain good estimates of $\Delta y^c$. The reason is that one cannot be confident that the coefficients reported in Table 1 would remain valid if the specification were changed slightly.

Another method for evaluating the accuracy of trends is to study autoregressions of fitted trend growth and analyze whether their moving average representations fit the patterns in Figure 5. One difficulty with this analysis is that the number of lags that can be used in these autoregressions depends on the realization. I sought to use five lags, but reduced the number of included lags whenever the covariance matrix of independent variables was close to singular. Also, I neglected those sample realizations where the fitted autoregression had roots above 0.999. Averaging across the remaining samples, I found that shocks that raise output growth initially are followed by further growth in output. In these estimates, half the total output increase takes place within 36 quarters in the case of $m_{20}$, 39 quarters in the case of $m_{40}$, and 42 quarters in the case of $m_{60}$. As I discuss below, I also used the model to construct realizations for the changes in hours. These were positively correlated with the disturbances that lead to long-run output changes, and this too differs from what is observed in Figure 5. It follows from these observations that the variations in the rate of growth of output induced by technical progress are only imperfectly captured by smooth trends, even in the $m_{60}$ case. It remains an open question whether this fit can be improved significantly with better estimators or whether estimating the changes in GDP growth rates induced by smooth technical progress is intrinsically difficult.

V. VAR Analysis

A fairly common multivariate approach to extracting the component of output due to technological disturbances is to study a vector autoregression that includes output and a measure of the labor input (Blanchard and Quah, 1989; Gali, 1999; Francis and Ramey, 2002). In this section, I apply this technique to series generated by the model. Mostly, I do this to evaluate the ability of VARs to decompose GDP accurately into its two components as well as their ability to mimic the responses of output and hours to permanent disturbances.

The VAR analysis in this section also serves as a check on the model itself because one can compare the results of VARs applied to my model-generated series to those that have been obtained when VARs have been fitted to U.S. data. I check, in particular, whether permanent disturbances initially lead hours to fall as found by Blanchard and Quah (1989), Gali (1999), and Francis and Ramey (2002). Unlike the
implications of the model, the point estimates in these studies do not involve initial declines in output, though the standard errors tend to be large enough to include small declines as distinct possibilities.

To construct hours, I sum series that capture the responses of hours to \( e^i \) and \( e^r \). The response to \( e^i \) can be computed from (16), (17), and (18). To compute the response of hours to \( e^r \), one needs a theory of the sources of temporary output fluctuations. For purposes of illustration, I attribute all these fluctuations to variations in \( k^G \) while setting the average value of \( k^G \) to 1.4. I thus linearize (12), (13), and (14) to obtain the changes in both \( k^G \) and \( h^m \) that correspond to the fluctuations in output induced by \( e^r \). The composite hours series I consider, \( h^m_t \), can then be written as

\[
(28) \quad h^m_t = \Theta_h(L) e^r_t + \theta_h(L) e^i_t.
\]

The resulting series appear to be stationary, with the maximum eigenvalue in regressions on three lags of hours being below 0.89 in all three specifications. By contrast, regressions of the composite output series on a constant, a lag of output, and three lags of the rate of growth of output rarely reject the hypothesis that these series have a unit root. In 5,000 replications, the Dickey-Fuller test rejected the existence of a unit root in only 67 cases for \( m_{m20} \), 38 cases for \( m_{m40} \), and 20 cases for \( m_{m60} \). By contrast, output growth appears stationary. Combining (26) and (27) output growth is given by

\[
(29) \quad \Delta y^m_t = \Theta_y(L) e^r_t + (1 - L) \theta_y(L) e^i_t.
\]

In 5,000 replications, the maximum eigenvalue in a regression of output growth on three lags was below 0.7 in all three specifications.

I thus consider bivariate VARs based on the composite series for the level of hours and the growth rate for output. Whether the inverse of such a VAR leads to accurate estimates of (29) and (28) depends in part on whether the representation given by these two equations is fundamental or not. If it is not fundamental, one cannot recover the \( e^r \)’s from convergent sums of past observations of \( \Delta y^m \) and \( h^m \). It is worth noting that, even if it is fundamental, many lags might have to be included to recover the true \( e^r \)’s so that a detrending method that relies on both past and future values of output growth to obtain the current estimate of the trend (as is true of the detrending method considered above) might be superior.

Lippi and Reichlin (1993) present a model where technological progress takes five quarters to diffuse and where the moving average representation implied by the model is not fundamental. By contrast, the one-sector model considered here implied that the roots of the determinant \( \Theta_y(L) \theta_y(L) - \Theta_h(L)(1 - L) \theta_y(L) \) are all greater than one, so the representation is fundamental.

I estimate VARs with three lags because these fit well in a sample of realized series with which I experimented. In state space form this estimated dynamic system is

\[
(30) \quad z_t = A z_{t-1} + \epsilon_Y^t
\]

where \( z_t = \{ \Delta y^m_t, h^m_t, \Delta y^m_{t-1}, h^m_{t-1}, \Delta y^m_{t-2}, h^m_{t-2} \} \), the first two rows of \( A \) are the estimated VAR coefficients and only the first two elements of \( \epsilon_Y^t, \epsilon_Y^{v1} \) and \( \epsilon_Y^{v2} \), are nonzero.

Unfortunately, in a small fraction of cases, the largest eigenvalue of \( A \) is greater than or equal to one. In 5,000 iterations of the model, this occurred 7 times for \( m_{m20} \), 48 times for \( m_{m40} \), and 69 times for \( m_{m60} \). I ignore these particular realizations in the remaining analysis. For the others, I compute the linear combination of \( \epsilon_Y^{v1} \) and \( \epsilon_Y^{v2} \) such that, for any value of this combination, the expectation of the long-run effects of \( \epsilon_Y^{v1} \) exactly cancel the expectation of the long-run effects of \( \epsilon_Y^{v2} \). I denote this linear combination by \( \epsilon_Y^* \) and focus first on the linear combination of disturbances that is orthogonal to \( \epsilon^*_Y \). Figure 7 reports the average response of hours and the level of output to the expectations of \( \epsilon_Y^{v1} \) and \( \epsilon_Y^{v2} \) induced by this permanent disturbance. The shock I consider is normalized so that it increases the permanent level of output by one unit.

The figure shows that, for all three moduli of

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20 I obtained very similar results, though the measurements of the cycle were slightly less accurate, by using a VAR with the rate of growth of productivity and the level of hours. In this alternate VAR, the permanent shock was orthogonal to the shock that had no long-run effect on productivity, as in Gali (1999) and Francis and Ramey (2002).
the complex roots of (24), both output and hours fall on impact. The latter is consistent with point estimates from VARs based on U.S. data, while the latter is not.\textsuperscript{21} It should be noted, however, that the estimated responses vary significantly from replication to replication. Thus, while the average initial response of output in the m₁ case is -0.63, the standard error across replications is nearly 1.0. Thus, the estimation of small initial output increases in empirical VARs provides only weak evidence against the model.

Qualitatively, the short-run responses in Figure 7 match those in Figure 5. They differ in the details, however. In particular, the model predicts the biggest instantaneous reduction in output and hours in the m₆₀ case. By contrast, the VARs ascribe this to the m₉₀ case. This discrepancy may be due to the model’s prediction that the decline in hours and output below the steady state lasts longest in the m₆₀ case. The VAR may be better at capturing relatively long declines than sharp and short-lived ones.

Another difference between the responses measured by the VAR and the actual responses of the trend concerns the amount of time it takes before permanent shocks have half of their effect on output. According to the VAR, it takes 15 quarters for this to occur in the m₂₀ case and 22 quarters in the m₄₀ and m₆₀ cases. This is considerably shorter than the actual responses, particularly in the latter two cases, where the estimates from autoregressions of fitted trends come much closer to the mark.

Consistent with the discussion by Thomas F. Cooley and Mark Dwyer (1998), the bivariate VAR is thus unable to mimic perfectly the behavior of the trend and the cycle.\textsuperscript{22} It is still worth obtaining a measure of the accuracy of a trend-cycle decomposition implied by these VARs and compare it to the accuracy of the trends I computed earlier. One attractive decomposition of this sort is to use the realized values of εᵣ together with the impulse response to this shock constructed from (30) to obtain estimates of yᵣ as defined in (27). I construct such estimates by using impulse response functions to the temporary disturbance that are truncated after 16 quarters.\textsuperscript{23}

In 5,000 replications of the model, the resulting average MSEs of these estimates of yᵣ were 1.5e-4, 1.9e-4, and 2.4e-4 for the cases m₂₀, m₄₀, and m₆₀, respectively. The corresponding correlations between the measured and actual cycles were 0.927, 0.900, and 0.860, respectively. Compared with the results from (21), these measures of accuracy are better for m₂₀ and worse for m₄₀ and m₆₀. Thus, the smoothness of technological progress matters when ranking these two methods of obtaining cycles, perhaps because future information is more useful in constructing one’s current estimate of the trend when trends are smoother.

VI. Conclusion

Shocks that lead to gradual and prolonged increases in productivity do not generate perfectly smooth changes in output because they create wealth effects. However, one can explain

\textsuperscript{21} Using a different methodology, Basu et al. (1998) do find small (insignificant) declines in output for a year when a positive technology shock hits the economy.

\textsuperscript{22} One of their examples (pp. 70–72) is a model where technology diffuses relatively slowly, though they assume it diffuses fully within five quarters. Consistent with their model, VARs fitted to data generated with this model exhibit short-run declines in output when there is a positive technology shock. However, other aspects of the impulse responses appear inconsistent with the model used to generate the data.

\textsuperscript{23} An alternative is to equate the cycle with forecasted declines in output over various horizons, as in Rotemberg and Woodford (1996). This provides less accurate estimates of yᵣ, probably because technological disturbances also generate forecastable changes in output.
the size of long-term changes in U.S. GDP growth rates with only small shocks of this type. This means that fairly smooth paths for GDP result from shocks of this sort when their size is empirically plausible. As a result, smooth trends provide fairly good measures of the impact of technical progress on GDP and, by the same token, detrended GDP is mostly affected by disturbances whose effect is only transitory.

It might be possible to make the response of GDP growth to technical progress even smoother by considering less standard models. In particular, there may be scope for reducing the size of the shocks to perceived wealth that accompany innovations that diffuse slowly. One potential method for doing so is to change the way information about these innovations is released in the economy. In this paper, I have supposed that information about the permanent effect of these shocks is released all at once, at the moment when these shocks have their first, negligible impact. It may be more empirically appealing to imagine that this information is released gradually. Moreover, some individuals may know about the permanent effects of certain technical breakthroughs before others so that the wealth revisions of different individuals may be staggered over time. Particularly if the more informed individuals are unable to borrow against their future income, the result might be a very smooth response of GDP to these shocks.

More generally, this paper suggests that it would be worthwhile to explore the aggregate consequences of alternative assumptions about the appearance and spread of new technologies. In this paper, I have assumed that the ultimate effects of the innovations that appear on any given date are independently distributed over time and that each innovation evolves in the same way. One could, instead, consider the possibility that innovations are bunched over time, so that the size of the innovations that appears on any given date depends on the size of innovations that appear on nearby dates. Along the same lines, shocks to technology at a point in time may alter the way past innovations diffuse either by leading old innovations to be discarded or by accelerating their spread. This would mean that, for the same contemporaneous effect, different shocks would have different effects on the future productivity of capital. These variations, which seem potentially important, are left for further research.

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