Speculative attacks on target zones

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1 The basic model

We consider a basic log-linear monetary model of the exchange rate. The exchange rate at any point in time is determined by

\[ s = m + v + \gamma E[ds]/dt \]  

(1)

where \( s \) is the log of the price of foreign exchange, \( m \) the log of the money supply, \( v \) a money demand shock term (incorporating shifts in real income, velocity, etc.), and the last term captures the effect of expected depreciation.

Money demand is assumed to follow a random walk with drift:

\[ dv = \mu dt + \sigma dz \]  

(2)

As Miller and Weller (1989) have shown, more complex processes, notably autoregressive ones, can be incorporated into the analysis without changing the qualitative results. We stick with this process for simplicity.

The general solution to the model defined by (1) and (2) for a fixed money supply has by now become familiar (see for example Froot and Obstfeld, 1989). It takes the form

\[ s = m + v + \gamma_1 + A \phi_1^v + B \phi_2^v \]  

(3)

where \( \phi_1 \) and \( \phi_2 \) are parameters that will be determined in a moment, and \( A \) and \( B \) are free parameters that need to be tied down by the economics of the situation.

To determine \( \phi_1 \) and \( \phi_2 \), we first note that by applying Ito’s Lemma we have

\[ E[ds]/dt = \mu + \mu_1[A \phi_1^v + \phi_2^v] + \frac{\sigma^2}{2} [\phi_1^v \phi_2^v + \phi_2^v \phi_1^v] \]

(4)

Substituting (4) back into (1), and comparing it with (3), we find that the roots are

\[ \phi_1 = \frac{-\gamma_1 + \sqrt{\gamma_1^2 + 2\gamma_1\\sigma^2}}{\gamma_1} > 0 \]

\[ \phi_2 = \frac{-\gamma_1 + \sqrt{\gamma_1^2 + 2\gamma_1\\sigma^2}}{\gamma_1} < 0 \]

(5)

We can now turn to the economic interpretation of (3). The first three terms in (3) evidently represent a sort of ’fundamental’ exchange rate: they reflect the combination of money supply, money demand, and the known drift in money demand. The other terms represent a deviation of the exchange rate from this fundamental value.

Suppose that the money supply were expected to remain unchanged at its initial level forever. Notice that \( v \) can take on any value. It seems reasonable to exclude solutions for the exchange rate that deviate arbitrarily far from the fundamental level when \( v \) takes on large positive or negative values. Thus under a pure float, in which the monetary authority is expected to remain passive whatever the exchange rate may do, we may assume \( A = B = 0 \). The exchange rate equation under a pure float is therefore

\[ s = m + v + \gamma_1 \mu \]

(6)

2 An exchange rate target with ‘small’ reserves

Now let us suppose that the monetary authority, instead of being passive, attempts to place an upper limit on the price of foreign exchange. Specifically, the monetary authority is willing to buy foreign exchange in an unsterilized intervention, up to the limit of its reserves, when the exchange rate goes above some level \( s_{max} \).

Provided that these reserves are small enough (we will calculate the critical size below), this attempt will lead to a speculative attack in which the whole of the reserves are suddenly exhausted when the exchange rate reaches \( s_{max} \).

We start by defining the initial money supply as the sum of reserves and domestic credit:

\[ m = ln(D + R) \]

(7)

Following the speculative attack, the money supply will fall to

\[ m' = ln(D) \]

(8)

Figure 8.1 illustrates the equilibrium before and after the speculative attack. After the attack, the exchange rate will be freely floating, with money supply \( m' \); so the post-attack exchange rate equation is

\[ s = m' + v + \gamma_1 \]

(9)

shown in Figure 8.1 as the locus \( F'F' \).

The attack will occur when \( v \) reaches the level at which the reduction in the money supply that results from the attack validates itself, by leading to the exchange rate \( s_{max} \); this is shown in Figure 8.1 as point \( C \), and corresponds to the level of \( v, v' \), such that
attack lies everywhere below the free float relationship \( FF \) corresponding to the initial money supply \( m \).

When reserves are small, then, the monetary authority fails in its effort to enforce an exchange rate target. The knowledge that it will try supports the currency; but eventually the target is overrun by a speculative attack. Notice that 'smooth pasting' nowhere makes its appearance in this analysis. Indeed, the pre-attack schedule in Figure 8.1 is not tangent to the exchange rate target.

Our next step is to enlarge the monetary authority's reserves, and show that if these reserves are sufficiently large, a 'smooth-pasting' solution emerges.

### 3 A target zone with large reserves

A variety of alternative potential speculative attack scenarios can be generated by varying the parameter \( A \) in equation (11). In Figure 8.2 we show the curves traced out by increasingly negative values of \( A \). A small absolute value of \( A \) corresponds to a speculative attack at \( C_1 \). A larger absolute value of \( A \) would produce an attack somewhere to the right of \( C_1 \), and this attack would consume more reserves because the implied fall in the money supply - measured as the horizontal distance from the attack point to the free float locus - would be larger.

It is immediately apparent, however, that one cannot in this way generate arbitrarily large speculative attacks. The reason is that the family of curves corresponding to different (negative) values of \( A \) all turn downward at some point, and for a sufficiently negative \( A \) the maximum of the curve lies below \( s_{\text{max}} \). But it is not possible for the exchange rate pre-attack to lie on a locus that passes above \( s_{\text{max}} \) before the attack takes place, since that would trigger the central bank's intervention.

The upshot is that the analysis of the previous section is valid only if the size of reserves is not too large; specifically, if the free float locus corresponding to the money supply that would follow elimination of all reserves does not lie to the right of the point \( C_2 \) in Figure 8.2.

If reserves are larger than this level, what must happen is that the pre-attack exchange rate equation is precisely that which leads to \( C_2 \). That is, \( A \) must be chosen so that the exchange rate locus is tangent to the target. Smooth pasting therefore emerges, not as the general solution of this model, but as its solution when the central bank's reserves are sufficiently large.

We can derive the critical level of reserves as follows. First, the exchange rate locus must be flat at \( v' \):

\[
\frac{ds}{dv} = 1 + a Ae^{i \nu'} = 0
\]

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Please note that the diagram and the text are from a page discussing speculative attacks on target zones. The text explains the dynamics of speculative attacks on exchange rates and the conditions under which a smooth-pasting solution emerges. It also introduces the concept of a target zone with large reserves, where the analysis is valid only if the size of reserves is not too large. The equation \( \frac{ds}{dv} = 1 + a Ae^{i \nu'} = 0 \) is used to derive the critical level of reserves for a smooth pasting solution.
\[ m' - m = -\frac{\mu}{a_1} \]  

But the change in the money supply in a speculative attack depends on the ratio of reserves to domestic credit:

\[ m' - m = -\ln \left( \frac{D + R}{D} \right) = -\ln \left( 1 + \frac{R}{D} \right) \]  

So the nature of the equilibrium changes from speculative attack to smooth pasting when

\[ \frac{R}{D} > e^{\mu} - 1 \]  

When this criterion is met, the central bank is able to hold the line at \( s_{\text{max}} \) with an infinitesimal intervention that slightly reduces the money supply, shifting the relationship between \( v \) and \( s \) down. If \( v \) then falls again, the exchange rate retreats down this new schedule; if \( v \) rises, another intervention must take place. These successive interventions would gradually shift the exchange rate schedule to the right. As long as the reserves remain sufficiently large, \( s_{\text{max}} \) will act as a reflecting barrier for the exchange rate, which will sometimes rise to \( s_{\text{max}} \), sometimes fall below it.

It is immediately apparent, however, that this process cannot go on indefinitely. When \( v \) is high, the monetary authority loses reserves; when \( v \) falls again, it does not regain them. So there is a gradual loss of reserves, which will gradually shift the exchange rate schedule to the right. Eventually the level of reserves will fall to the critical level where a speculative attack becomes possible; at that point, the next time that the exchange rate drifts up to the level \( s_{\text{max}} \) there will be a full-scale speculative attack that eliminates all remaining reserves. In other words, a country that starts with large reserves will go through a 'smooth-pasting' phase where small interventions succeed in holding the line on the exchange rate; but there will be a gradual (albeit intermittent) drain on reserves, and as in conventional speculative attack models there will eventually be a crisis once reserves have dropped to a critical level.

This is not a very complicated analysis. Nonetheless, it makes several points that have been obscured in some of the recent literature on the subject.

First, it is clear from this model that looking at the case of bounded fundamentals is not equivalent to looking at the case of an exchange rate target. If we were to use the bounded fundamentals technique on this model, we would replace the idea of a target on \( s \) with that of an upper limit on \( m + v \). This would correctly capture the notion of what happens.
as long as reserves are sufficiently large to achieve the smooth-pasting solution; but it would miss both the case where initial reserves are small, and the logic of eventual crisis.

Second, in a related point, smooth pasting is not a general result of this model, the way it appears to be in the bounded fundamentals formulation. On the contrary, it is a special case that obtains when reserves are sufficiently large; otherwise the logic is that of speculative attack: the present exchange rate is tied down by the requirement that there be no foreseeable jumps in the future exchange rate.

Third, this model helps settle a controversy about the justification for the smooth-pasting result. Some economists approaching the problem from the perspective of optimization models have questioned the use of the smooth-pasting condition in ad hoc monetary models of this kind, arguing that a condition that arises from optimization is hard to justify when optimizing behaviour is at best implicit. Those of us doing the ad hoc models have argued on the contrary that the condition can equally be seen as being implied by arbitrage.1 In this model we see smooth pasting emerge as the limit of the 'no foreseeable jumps' condition of a speculative attack model – essentially an arbitrage condition – when reserves are sufficiently large.

4 A gold standard model

In the remainder of this paper we make use of the type of analysis developed in earlier sections to attack a particular problem that has been the subject of several recent papers, that of the role of speculative attacks under a gold standard system.

Several papers, notably Buitier (1989) and Grilli (1989), have analysed the problem of speculative attack in a gold standard model. Grilli implements the model empirically as well. However, as we will show shortly, straightforward application of the standard speculative attack model to the problem of a gold standard runs into serious problems. The standard model, in its simplest version, seems to suggest that there will in fact be no speculative attacks on a gold standard, that such a regime will end with a whimper rather than a bang; this runs counter to both intuition and experience. Worse yet, with a little elaboration one runs into a serious conceptual paradox that undermines the logic of the analysis. What we will do in this part of the paper is show how an economically reasonable model of speculative attacks on a gold standard can be created by treating such a standard as a boundary between two imperfectly sustainable target zones.

The basic gold standard model may be presented as a two-country version of the model at the beginning of this paper. The exchange rate depends on the ratio of two countries' money supplies, a demand shock term, and the expected rate of depreciation:

\[ s = m - m^* + v + \gamma \frac{E[ds]}{dt} \]  

(19)

Each country's money supply consists of domestic credit plus reserves:

\[ m = \ln(D + R) \]  

(20)

\[ m^* = \ln(D^* + R^*) \]

Reserves, however, are now taken to consist of gold, which is in fixed world supply:

\[ R + R^* = G \]  

(21)

As before, we need to specify a process for the money demand term. We will initially suppose that it is a simple random walk without drift – i.e., \( \mu = 0 \). The implications of more complex stochastic processes are discussed below.

We suppose that the monetary authorities of the two countries stand ready to buy or sell gold to maintain fixed prices of their currencies in terms of gold, and hence in terms of each other; the implied exchange rate is \( s_{par} \). This regime will continue until one country or the other runs out of gold.

The seemingly obvious assumptions are that as long as the regime is in effect, there will be no expected change in exchange rates; and that when the regime collapses, the exchange rate reverts to a free float. It turns out, however, that this combination of assumptions yields the economically implausible result that there are no speculative attacks.

To see why, first ask how reserves would appear to evolve if we in fact assume \( E[ds]/dt = 0 \). Then when \( v \) rises, gold will flow from the first country to the second; when it falls, it will flow in the other direction. Ignoring the possibility of speculative attack, this process could continue until the ratio of money supplies reaches either a maximum or minimum value. The maximum value of \( m - m^* \) occurs when all gold has flowed out of the first country; at that point we have

\[ m = \ln(D) \]  

(22)

and

\[ m^* = \ln(D + G) \]  

(23)

Similarly, \( m - m^* \) reaches a minimum when all the gold has flowed to the first country, so that
\[ m = \ln(D + G) \]  
(24) 

and 

\[ m^* = \ln(D) \]  
(25) 

In the conventional speculative attack literature, we show the necessity of a speculative attack by noticing that if agents were naive, and did not anticipate the possibility of regime collapse, there would be a foreseeable capital gain or loss at the moment of transition. Suppose, then, that agents were naive, and did not realize that a regime change was in prospect. Would they be missing a profit opportunity? Under the assumption of naivete, the gold standard would last until reserves of one country or another run out. Let us suppose that it is the first country that runs out of gold; it would run out at a level of \( v \), determined by the condition 

\[ s_{par} = m - m^* + v \]  
(26) 

If the exhaustion of the country's gold is followed by a transition to pure floating, the exchange rate following the transition would be determined by 

\[ s = m - m^* + v \]  
(27) 

But by comparing (26) and (27) we find that 

\[ s = s_{par} \]  
(28) 

That is, there is no jump in the exchange rate. This implies that there need not be any speculative attack. 

This is an economically implausible conclusion. Matters become even worse if the process determining \( v \) is not a simple random walk — if it has drift, or autoregression. In that case one arrives not simply at an implausible result but at a paradox: under some conditions a country may run out of reserves under a fixed rate before it meets the usual criterion for a speculative attack. This 'gold standard paradox' has been the subject of several recent papers (Krugman and Rotemberg, 1990; Buitser and Grilli, 1989). However, in this paper we focus only on the case of a random walk, in which there is not strictly speaking a paradox, simply an implausible result. 

What we show next is that a much more satisfactory result emerges if we view a gold standard not as a one-time regime that is gone when once it has collapsed, but instead as a regime that is reinstated when feasible. In this case, as we will see, the gold parity becomes a boundary between two target zones.

5 Gold parity as a boundary

The analysis of speculative attacks on a gold standard can be made much more plausible if we make one assumption that is slightly different from the usual speculative attack setup.

The necessary assumption is the following: central banks do not give up when they run out of gold. Instead, they remain willing to buy gold at the par value, and thus to reinstate a gold standard if the opportunity arises.

An example may convey the essence of this assumption. Suppose that our two countries are America and Britain, and that they have established par values of gold of $35 and £7 per ounce. If both countries have positive gold reserves, this will peg the dollar-pound exchange rate at 5. Suppose, however, that America has run out of gold. Then the exchange rate may float above this level — say, at £7 per pound. The price of gold will be set by the willingness of the British central bank to sell it, at £7 per ounce.

What we will assume is that even though America has run out of gold, its central bank still remains willing to buy gold if the price falls to 35 dollars. (It would be willing to sell gold at that price also, but it doesn't have any to sell). With an exchange rate of 7, of course, the price of gold is $49, so there will be no current sales; but if the exchange rate falls (the dollar appreciates) to 5, gold purchases will commence.

Conversely, if Britain has run out of gold, the exchange rate will float at a level below 5; but if it rises to 5, Britain's central bank will again buy gold.

Consider what this implies. If the exchange rate is above 5, then everyone knows that if it falls to 5 America will buy gold and Britain sell it — which means that America will increase its money supply and Britain reduce its money supply. This means that when America is out of gold, and the exchange rate is floating, the float is not free. Instead, there is in effect a one-sided target zone, in which there is a de facto commitment to support the pound with sterilized intervention if the dollar strengthens too much.

The reverse is also true: when Britain has run out of gold, the float is in effect a target zone with a commitment to support the dollar with unsterilized intervention if the dollar strengthens to its par value.

This tells us that the par value implied by the prices at which each currency is pegged to gold may be regarded as a boundary between two one-sided target zones. In the lower zone, in which America has all the gold — which we will call the A-zone — the dollar-pound exchange rate is held below its free-float locus by the prospect of US gold sales and British
Figure 8.3 A gold standard with large reserves

gold purchases if the pound rises too much. In the B-zone, where Britain has all the gold, the rate is correspondingly held above its free-float locus.

If the world's gold stock is large enough, the picture looks like Figure 8.3, which plots the exchange rate against \( v \). (We will describe the case with insufficient gold backing for the world's currencies below.) The lines \( F_A F_D \) and \( F_B F_D \) represent the free-float loci — that is, \( F_A F_D \) represents how \( s \) would vary with \( v \) if America had all the gold and there was no prospect of future intervention, and \( F_B F_D \) the corresponding case with all gold in British hands. The actual relationship in the A-zone, however, is that which we have already seen for a one-sided target zone with large reserves: a curve that lies below the free-float locus and is tangent to the par value line at some value \( v_A \). Similarly, in the B-zone the relationship between \( v \) and \( s \) lies above the free-float locus and is tangent to the par value line at \( v_B \).

The relationship between \( v \) and \( s \) is therefore indicated by the curve on the left up to \( v_A \); the par value is sustained between \( v_A \) and \( v_B \); and \( s \) follows the curve on the right for \( v \) greater than \( v_B \). Outside the range where the par value is sustained, the prospect of a return to the gold standard either supports or depresses the exchange rate.

What happens if \( v \) starts within the range where the par value can be sustained, then drifts out of that range, say to \( v_B \)? The answer is that as long as we are on the 'flat', there will be a gradual American loss of gold. When \( v_B \) is reached, however, there will be a speculative attack that leads to a discrete American loss of its remaining gold. The reason is that the post-attack \( v - s \) relationship is convex, so that the variance term makes \( E[(d\varepsilon)/dt]\) positive. That is, when the gold standard collapses the expected rate of dollar depreciation immediately goes from zero to some positive number, reducing relative American money demand — even if \( v \) is expected to fall. Similarly, if \( v \) drops to the bottom of the range there will be a speculative attack that leads to a discrete British loss of its remaining gold.

The reason why the currency of the country that runs out of gold is expected to depreciate immediately following the gold exhaustion is somewhat ironic: it is the result of the expectation that the country will try to buy gold if its currency should subsequently appreciate to the par value, which therefore depresses its value under the float.

What happens if America has no gold, and \( v \) drifts back into the range in which the par value is enforced? The answer is that there is a speculative run into the dollar, leading to a discrete gain in reserves at British expense. It may be useful to illustrate this model of the gold standard more explicitly, retaining the assumption that \( v \) follows a random walk (although Figure 8.3 remains valid even when \( v \) follows more complex processes; see Krugman and Rotemberg, 1990).

We begin by noting that when \( v \) follows a random walk with no drift, the two roots in the solution sum to zero. Thus the basic exchange rate equation may be written

\[
s = m - m^* + v + A e^{\alpha v} + B e^{-\alpha v}
\]

where \( \alpha \) may be calculated using the methods of Section 1.

There are now two de facto target zones: the 'A-zone' in which America has all the gold, and the 'B-zone' in which Britain has all the gold. The relative money supplies in these zones are therefore as follows: in the A-zone,

\[
m - m^* = \ln \left( \frac{D + G}{D^*} \right)
\]

while in the B-zone

\[
m - m^* = \ln \left( \frac{D}{D^* + G} \right)
\]
To calculate \( v_A \), we first note that since in the A-zone \( v \) is unbounded below, we must have \( B = 0 \), and must choose a value of \( A \) such that the exchange rate reaches its par value at \( v_A \):

\[
s_{par} = \ln \left( \frac{D + G}{D^*} \right) + v_A + Ae^{\alpha_s} (32)
\]

Also, the curve must be flat at \( v_A \):

\[
\frac{ds}{dt} = 1 + \alpha Ae^{\alpha_s} = 0 (33)
\]

Putting these together, we find that

\[
v_A = s_{par} - \ln \left( \frac{D + G}{D^*} \right) - \frac{1}{\alpha} (34)
\]

A similar calculation shows that

\[
v_B = s_{par} - \ln \left( \frac{D}{D^* + G} \right) - \frac{1}{\alpha} (35)
\]

What is the significance of the term \( 1/\alpha \)? It is the horizontal distance from each end of the gold standard range to the corresponding free float locus. It therefore measures the extent to which the target zone aspect of the exchange regime when one country has run out of gold leads to a collapse of the gold standard before the gold would have run out under a perfectly credible system. And \( 1/\alpha \) also measures the change in the log of the ratio of national money supplies that occurs when there is a speculative attack.

This example illustrates how a gold standard with limited gold reserves may be modelled as a boundary between two target zones. However, the example also reveals a problem. As drawn in Figure 8.3, we show \( v_B > v_A \), so that there is a range in which the par value can be maintained. But there is no guarantee that this is true. We note that

\[
v_B - v_A = \ln \left( \frac{D^* + G}{D} \right) + \ln \left( \frac{D + G}{D^*} \right) - \frac{2}{\alpha} (36)
\]

This will be positive only if gold reserves \( G \) are large enough relative to the world money supply. When gold reserves are sufficient, we get the story illustrated in Figure 8.3. But what if they are not sufficient?

On reflection, the story is apparent: it is illustrated in Figure 8.4. The par exchange rate \( s_{par} \) still represents the boundary between two target zone regimes, but the loci in each regime no longer 'smooth paste' to the par value. Instead, they smooth paste to each other at some critical value of \( \nu \). Whenever \( \nu \) crosses that value, there is a speculative attack that transfers all of the gold from America to Britain or vice versa. The central banks are trying to enforce a gold parity, but one or the other is always failing.

6 Conclusions

The literature on deterministic speculative attacks and the more recent literature on target zones share the insight that to understand how an exchange regime works, one must also understand how it ends. Capital flows under fixed rates depend critically on expectations of abandonment of parities. Exchange rates under floating may depend equally critically on expectations of future efforts to peg. These literatures therefore are closely related in spirit, and one would like to tie them together.

This paper offers one way to link the two views. Speculative attacks on target zones emerge in much the same way as speculative attacks on fixed rates, but the stochastic aspect of the model makes the analysis richer and, one hopes, adds insight. In particular, a target zone approach allows a much more satisfying analysis of speculation under a gold standard than is possible using the previous standard models.
NOTES

1 Dumas (1990) argues that since the tangency condition in these models does not arise from optimization, it really should not be called ‘smooth pasting’. This seems a semantic point, and in any case the terminology has already become so common that it really cannot be undone.

2 The equilibrium proposed here is similar, albeit with a rather different justification, to the solution to the gold standard paradox proposed by Delgado and Dumas (1990).

REFERENCES

(1989), ‘The linkage between speculative attack and target zone models of exchange rates’, NBER Working Paper No. 2918, and see their chapter in this volume.